TRANSFORMATIONS OF SERIES OF E-FUNCTIONS

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1. Introductory. The transformation [1, p. 25, 2, p. 369]

$$F\begin{pmatrix}\alpha, & \beta, & \gamma, & \delta, & -l; & 1\\ \alpha-\beta+1, & \alpha-\gamma+1, & \alpha-\delta+1, & \alpha+l+1\end{pmatrix} \tag{1}$$
$$=\frac{(\alpha+1;l)\left(\frac{1}{2}\alpha-\beta+1;l\right)}{\left(\frac{1}{2}\alpha+1;l\right)(\alpha-\beta+1;l)}F\begin{pmatrix}\alpha-\gamma-\delta+1, \frac{1}{2}\alpha, \beta, -l; 1\\ \alpha-\gamma+1, \alpha-\delta+1, \beta-\frac{1}{2}\alpha-l\end{pmatrix},$$

where l is a positive integer, is a special case of a formula of Whipple's. It, and other transformations of the same kind, can be employed to obtain transformations of series of E-functions. Two such transformations are:

$$\sum_{n=0}^{l}\frac{(\alpha;n)(\beta;n)(-l;n)}{n!(\alpha-\beta+l;n)(\alpha+l+1;n)}E\left\{\begin{matrix}p;\alpha_r\\ \Delta(m;\rho-n),\Delta(m;\sigma-n),\Delta(m;\alpha+\rho+n)\\ \Delta(m;\alpha+\sigma+n),\rho_1,\rho_2,\cdots,\rho_q\end{matrix}:z\right\}$$
$$=\frac{(\alpha+1;l)\left(\frac{1}{2}\alpha-\beta+1;l\right)}{\left(\frac{1}{2}\alpha+1;l\right)(\alpha-\beta+1;l)}\sum_{n=0}^{l}\frac{\left(\frac{1}{2}\alpha;n\right)(\beta;n)(-l;n)}{n!\left(\beta-\frac{1}{2}\alpha-l;n\right)}\left(\frac{2}{m}\right)^n \tag{2}$$
$$\times E\left\{\begin{matrix}\Delta(2m;\alpha+\rho+\sigma+n-1),\alpha_1,\alpha_2,\cdots,\alpha_p\\ \Delta(2m;\alpha+\rho+\sigma-1),\Delta(m;\rho),\Delta(m;\sigma),\\ \Delta(m;\alpha+\rho+n),\Delta(m;\alpha+\sigma+n),\rho_1,\rho_2,\cdots,\rho_q\end{matrix}:z\right\};$$

$$\sum_{n=0}^{l}\frac{(\alpha;n)(\beta;n)(-l;n)}{n!(\alpha-\beta+1;n)(\alpha+l+1;n)}E\left\{\begin{matrix}\Delta(m;\gamma+n),\Delta(m;\gamma-\alpha-n),\alpha_1,\cdots,\alpha_p\\ \Delta(m;\sigma+n),\Delta(m;\sigma-\alpha-n),\rho_1,\cdots,\rho_q\end{matrix}:z\right\}$$
$$=\frac{(\alpha+1;l)\left(\frac{1}{2}\alpha-\beta+1;l\right)}{\left(\frac{1}{2}\alpha+1;l\right)(a-\beta+1;l)}\sum_{n=0}^{l}\frac{(\sigma-\gamma;n)\left(\frac{1}{2}\alpha;n\right)(\beta;n)(-l;n)}{n!\left(\beta-\frac{1}{2}\alpha-l;n\right)(-m^2)^n} \tag{3}$$
$$\times E\left\{\begin{matrix}\Delta(m;\gamma),\Delta(m;\gamma-\alpha-n),\alpha_1,\cdots,\alpha_p\\ \Delta(m;\sigma-\alpha),\Delta(m;\sigma+n),\rho_1,\cdots,\rho_q\end{matrix}:z\right\}.$$

In these formulae m is a positive integer,

$$(\alpha;0)=1,\ (\alpha;m)=\alpha(\alpha+1)\cdots(\alpha+m-1), \tag{4}$$

and $\Delta(m;\alpha)$ denotes the set of parameters

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