ON A THEOREM OF FEJÉR

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1. Let

$$T\colon (\tau_{n\nu}) \qquad (n = 0, 1, 2, \cdots; \nu = 0, 1, 2, \cdots)$$

be an infinite Toeplitz matrix satisfying the conditions

$$\text{(i)} \qquad \lim \tau_{n\nu} = 0$$

for every fixed ν,

$$\text{(ii)} \qquad \lim \sum_{\nu=0}^{\infty} \tau_{n\nu} = 1$$

and

$$\text{(iii)} \qquad \sum_{\nu=0}^{\infty} | \tau_{n\nu} | \leqq K ,$$

K being an absolute constant independent of n.

Given a sequence (S_n) if

$$\lim \sum_{\nu=0}^{\infty} \tau_{n\nu} S_\nu = S ,$$

then we say that the sequence (S_n) or the series with partial sums S_n is summable (T) to the sum S.

2. Suppose that $f(x)$ is integrable in the Lebesgue sense and periodic with period 2π. Let

$$f(x) \sim \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) .$$

Let

$$\sum_{n=1}^{\infty} n(b_n \cos nx - a_n \sin nx) = \sum B_n(x)$$

be the derived series of the Fourier series of $f(x)$. Fixing x, we write

$$\psi_x(t) = f(x + t) - f(x - t) .$$

Fejér [1] has proved the following

Received July 20, 1960.