PACIFIC JOURNAL OF MATHEMATICS
Vol. 15, No. 3, 1965

TOPOLOGIES FOR LAPLACE TRANSFORM SPACES

DONALD E. MYERS

In this paper four topologies are compared:
(i) an L_2-type topology on the space of functions having bilateral transforms,
(ii) an L_1 and (iii) an L_2-type topology on the space of transforms, and
(iv) finally that of one form of convergence of compact subsets for the space of analytic functions. It is shown that sequential convergence in (i) implies (iii) and (iv) and (ii) implies (i) and (iv) and hence (iii).

In an earlier paper [2]; the author used equivalence classes of analytic functions to construct an imbedding space for Schwartz Distributions. The mechanism for constructing the mapping was the bilateral Laplace Transform, in this way the traditional approach to operational calculus was preserved. In that paper a topology was imposed on the imbedding space from the space of analytic functions. We now obtain some additional results about the possible topologies defined on the space of analytic functions.

THEOREM 1. *Let $F_j(t), j = 0, 1, 2, \cdots, -\infty < t < \infty$ be real valued functions such that for each j*

$$d_1(F_j) = \left[\int_0^\infty |\, e^{-\sigma_1 t} F_j(t)\,|^2\, dt\right]^{1/2} < \infty$$

and

$$d_2(F_j) = \left[-\int_0^{-\infty} |\, e^{-\sigma_2 t} F_j(t)\,|^2\, dt\right]^{1/2} < \infty$$

where $-\infty < \sigma_1 < \sigma_2 < \infty$. If

$$d(F_j - F_0) = d_1(F_j - F_0) + d_2(F_j - F_0) \rightarrow 0$$

as $j \rightarrow \infty$ then

(i) *$f_j(z) \rightarrow f_0(z)$ uniformly on compact subsets of $\sigma_1 < R(z) < \sigma_2$ where $f_j(z) = \int_{-\infty}^{\infty} e^{-zt} F_j(t) dt$*

(ii) $\|\, f_j - f_0 \,\|_x \rightarrow 0$

where

Received April 24, 1964. This research was partially supported by NSF Contract Grant NSF GP-1951.