PACIFIC JOURNAL OF MATHEMATICS
Vol. 17, No. 2, 1966

METRIZABILITY AND COMPLETENESS IN NORMAL MOORE SPACES

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B. Fitzpatrick, Jr. and D. R. Traylor proved [Theorem 1, Pac. J. Math., to appear] that if there is a normal, nonmetrizable Moore space then there is one which is not locally metrizable at any point. The primary purpose of this paper is to extend the stated result to include normal, complete Moore spaces. That is, it is established that there is a normal, complete, Moore space which is not locally metrizable at any point, provided there exists a normal, complete, nonmetrizable Moore space. Indeed, it is further established that, provided there exists a nonmetrizable, normal, complete Moore space, then there is one which is also connected, locally connected, not locally metrizable at any point, and, using a result of Younglove's [Theorem 1, "Concerning metric subspaces of nonmetric spaces," Fund. Math., 48 (1949), 15–25], which contains a dense metrizable subset.

F. B. Jones [Bull. Amer. Math. Soc. 43 (1937), 671–677] showed that if $2^{\aleph_0} < 2^{\aleph_1}$., then every normal separable Moore space is metrizable. It is established in this paper that if each normal, separable, connected space satisfying Axioms 0, 1, and 2 of [R. L. Moore, Foundations of Point Set Theory, Amer. Math. Soc. Colloq. Pub. No. 13 Providence, R. I. 1962] is metrizable, then each normal separable Moore space is metrizable.

Other theorems of this ilk are included in this paper.

The statement that S is a Moore space means that there exists a sequence of collections of regions satisfying Axiom 0 and the first three parts of Axiom 1 of [5]. Such a sequence is said to be a development and should a Moore space have a development which also satisfies the fourth part of Axiom 1 of [5], that space is said to be a complete Moore space.

All other definitions and terms are as in [1, 2, 3, 4, 5, 6, 7].

LEMMA. *If each normal, connected, locally connected Moore space is metrizable, then each normal Moore space is metrizable.*

Proof. The proof is similar to that used to establish Theorem 3 of [6].

Received November 1, 1964. Theorem 1 and Theorem 4 were presented to the American Mathematical Society August 27, 1964.