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A CLASS OF BISIMPLE INVERSE SEMIGROUPS[1]

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The purpose of this paper is to study a certain generalization of the bicyclic semigroup and to determine the structure of some classes of bisimple (inverse) semigroups mod groups.

Let S be a bisimple semigroup and let E_S denote the collection of idempotents of S. E_S is said to be integrally ordered if under its natural order it is order isomorphic to I^0, the nonnegative integers, under the reverse of their usual order. E_S is lexicographically ordered if it is order isomorphic to $I^0 \times I^0$ under the order $(n, m) < (k, s)$ if $k < n$ or $k = n$ and $s < m$. If $\mathscr{H}$ is Green's relation and E_S is lexicographically ordered, $S/\mathscr{H} \cong (I^0)^4$ under a simple multiplication. A generalization of this result is given to the case where E_S is n-lexicographically ordered. The structure of S such that E_S is integrally ordered and the structure of a class of S such that E_S is lexicographically ordered are determimed mod groups. These constructions are special cases of a construction previously given by the author. This paper initiates a series of papers which take a first step beyond the Rees theorem in the structure theory of bisimple semigroups.

The theory of bisimple inverse semigroups has been investigated by Clifford [2] and Warne [7], [8], and [9].

If S is a bisimple semigroup such that E_S is lexicographically ordered, $S/\mathscr{H}$ is shown to be isomorphic to the semigroup obtained by embedding the bicyclic semigroup C in a simple semigroup with identity by means of the Bruck construction [1]. We denote this semigroup by CoC. An interpretation of this construction introduced by the author in [10] is used.

In [2, p. 548, main theorem], Clifford showed that S is a bisimple inverse semigroup with identity if and only if $S \cong \{(a, b)\colon a, b \in P\}$, where P is a certain right cancellative semigroup with identity isomorphic to the right unit subsemigroup of S, under a suitable multiplication and definition of equality. In the special case $\mathscr{L}$ (Green's relation) is a congruence on P(equivalently, $\mathscr{H}$ is a congruence on S), Warne showed [8, p. 1117, Theorem 2.1; p. 1118, Theorem 2.2 and first remark] that $P \cong U \times P/\mathscr{L}$, where U is the group of units of P (of S), under a Schreier multiplication or equivalently, $S \cong \{((a, b), (c, d))\colon a, c \in U,\ b, d \in P/\mathscr{L}\}$. Warne also notes [8, p. 1118, second remark and p. 1121, Example 2] that a class of semigroups

[1] Some of the results given here have been stated in a research announcement in the Bull. Amer. Math. Soc. [12].