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RELATIVE GENERAL POSITION

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In this paper we will say that a piecewise linear map $f: K \to M$ from a finite complex into an n-manifold is a general position (gp) map, if for every pair of simplexes, A, B, contained in K,

$$\text{(dimension of the singularities of } f \mid A + B) \leqq \text{(dimension of } A) + \text{(dimension of } B) - n\,.$$

By letting $B = \emptyset$, we see that a gp map into an n-manifold is an embedding on each simplex of dimension less than or equal to n. Also note that the restriction of a gp map to a subcomplex is again a gp map. It is well known that every map f of a complex into a combinatorial manifold can be homotopically approximated by a gp map, g, on some subdivision of the complex. One might suppose that, if L is a subcomplex on which f is already a gp map, then $g \mid L$ could be made equal to $f \mid L$. However, this cannot be done, in general, even if the manifold is a Euclidean space and the complex is a subdivision of a cell. (See the Remark at the end of § 3.)

In § 3 are two general position theorems which fix the map on a subcomplex on which it is already a gp map, but not without some severe restrictions. These theorems are stated in terms of relative general position (rgp) which applied to maps from a pair into a pair. Section 4 considers maps $f: (D, \text{Bd } D) \to (M, N)$ of a 2-manifold, D, into a 3-manifold, M, with 2-submanifold, N, with the added restriction that $f(\text{Bd } D) \cdot f(D - \text{Bd } D) = \emptyset$. It is, in general, impossible in this setting to make f into an rgp map while keeping $f \mid \text{Bd } D$ fixed. However, two "relative normal position theorems" are proved which make the singularities "nice" while not considering a particular subdivision.

The proofs are contained in § 5 through 8.

It should be pointed out that E. C. Zeeman's definition of general position (see [9], p. 59, for general description and [10], Chapter 6, for detailed discussion and proofs) differs from the one used here and avoids most, if not all, of the difficulties encountered in this paper. Thus in a round-about fashion this paper points up several advantages in Zeeman's definition. However, Zeeman's definition may be undesirable for certain purposes. The main difference between the definitions is that Zeeman cannot require that a general position map from a complex K into a manifold to be *both* in general position on each subcomplex of K *and* a homeomorphism on each simplex of K.