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SAMPLE FUNCTION BEHAVIOR OF INCREASING PROCESSES WITH STATIONARY, INDEPENDENT INCREMENTS

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In this paper we are concerned with the sample functions of increasing stochastic processes, X_ν, having stationary, independent increments; normalized so that X_ν has no deterministic linear component and $X_\nu(0) = 0$, (i.e., X_ν is a subordinator).

For h a fixed function, we are interested in the following two events:

$$\{\omega\colon X_\nu(t, \omega) > h(t) \text{ infinitely often as } t \to 0\} ,$$
$$\{\omega\colon X_\nu(t, \omega) > h(t) \text{ infinitely often as } t \to \infty\} .$$

In case X_ν is a stable process, Khinchin has given integral tests to apply to a wide class of h's in order to decide whether one, the other, or both of these two events have probability zero or one. The purpose of this paper is to give similar results, without assuming X_ν to be stable.

We also prove (Theorem 3) a variation-type theorem concerning the sample functions. Theorem 4 is an L_1 convergence theorem for the distribution function as time goes to zero.

2. **Notation.** We let ν be a measure on $(0, \infty)$ with

$$\int_0^\infty y(1 + y)^{-1}\nu(dy) < \infty .$$

Then, we let

$$g_\nu(u) = \int_0^\infty (1 - e^{-uy})\nu(dy) ;$$
$$\varphi_\nu(t, u) = \exp(-tg_\nu(u)) .$$

We let X_ν be a function of two variables; the first variable being an element in $[0, \infty)$ and the second variable being an element in a probability space Ω with probability measure P. We take $X_\nu(0, \omega) = 0$ for all $\omega \in \Omega$, and we take $X_\nu(\cdot, \omega)$ to be an increasing right continuous function (The range of X_ν is taken to be a subset of $[0, \infty)$.). We require that

$$\int_0^\infty e^{-ux} d_2F_\nu(t, x) = \varphi_\nu(t, u)$$

where