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STABILITY IN TOPOLOGICAL DYNAMICS

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This paper is concerned with two types of stability in transformation groups. The first is a generalization of Lyapunow stability. In the past this notion has been discussed in a setting where the phase group was either the integers or the one-parameter group of reals. In this paper it is defined for replete subsets of a more general phase group in a transformation group. Some connections between this type of stability and almost periodicity are given. In particular, it is shown that a type of uniform Lyapunov stability will imply Bohr almost periodicity. The second type of stability in this paper is a limit stability. This gives a condition which is necessary and sufficient for the limit set to be a minimal set. Finally, these two types of stability are combined to provide a sufficient condition for a limit set to be the closure of a Bohr almost periodic orbit.

Throughout this paper X will be assumed to be a uniform space. It will be implicity assumed that the Hausdorff topology of X is the one induced by the uniformity. T will denote a topological group and the triple (X, T, π) will be called a transformation group provided X and T are as above and $\pi: X \times T \to X$ such that if e is the identity of T then:

(1) $\pi\,(x, e) = x$ for all x in X,

(2) $\pi\,(\pi\,(x, t_1), t_2) = \pi\,(x, t_1 t_2)$ for all x in X and t_1, t_2 in T,

(3) π is continuous. Henceforth we shall write $\pi\,(x, t) = xt$; and if $A \subset T$ then $xA = \{xt: t \in A\}$.

DEFINITION 1. A subset A of T is called $\{$*left*$\}\{$*right*$\}$ *syndetic* [6] in T provided there exists a compact set $K \subset T$ such that $\{AK = T\}$ $\{KA = T\}$. It is clear that if A is left syndetic in T then A^{-1} is right syndetic in T.

DEFINITION 2. A point $x \in X$ is called *S-Lyapunov stable* ($S \subset T$) with respect to a set $B \subset X$ provided that for each index α of X there exists an index β of X such that if $y \in B \cap x\beta$ then $yt \in xt\alpha$ for all t in S.

THEOREM 1. *If S is left syndetic in T and* Cl (xT) (= *closure* xT) *is compact then a necessary and sufficient condition that $x \in X$ be T-Lyapunov stable with respect to xT is that x be S-Lyapunov stable with respect to xT.*