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REFLECTION LAWS OF SYSTEMS OF SECOND ORDER ELLIPTIC DIFFERENTIAL EQUATIONS IN TWO INDEPENDENT VARIABLES WITH CONSTANT COEFFICIENTS

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In this paper we shall consider the reflection of solutions of systems of equations

$$u_{xx} + u_{yy} + Au_x + Bu_y + Cu = 0 \ , \tag{1·1}$$

where $u = (u_1, u_2, \cdots, u_n)^T, A, B, C$ are constant, pairwise commutative $n \times n$ matrices, across an analytic arc κ on which the solutions satisfy n analytic linear differential boundary conditions. If the boundary conditions have coefficients which are analyiic in a specific preassigned geometrical region cantaining κ, then we shall show that the solution of (1.1) satisfying such boundary conditions can be extended across κ, provided certain inequalities are satisfied. Moreover, the region into which u can be extended will depend only on the analytic arc κ, the original region, and the coefficients of the boundary conditions; i.e., we shall have reflection "in the large" and the region will not be restricted by the equation.

There are two basically different situations considered, the results of which are stated in Theorem 1, Theorem 2, and Theorem 3.

Theorem 1 treats the reflection of solutions of a system (1.1) initially given on an open set Ω for which the boundary conditions on an arc κ adjacent to Ω are

$$\sum_{\beta=1}^{n} p_{\alpha\beta}(D)u_\beta = f_\alpha(z) \ , \qquad \alpha = 1, 2, \cdots, n \text{ or } \kappa$$

where

$$p_{\alpha\beta}(D) = \sum_{r+s\leqq k<2n} < p^{rs}_{\alpha\beta}(z) D_x^r D_y^r$$

with $p^{rs}_{\alpha\beta}(z)$ and $f_\alpha(z)$ analytic in $\Omega \cup \kappa \cup \hat{\Omega}$, where $\hat{\Omega}$ is an open set determined by κ adjacent to κ and disjoint from Ω. In the event that two inequalities involving the $p^{rs}_{\alpha\beta}(z)(r + s = k)$ are satisfied, then we can reflect the solution of the system across κ into $\kappa \cup \hat{\Omega}$, so that the original solution is extended into all of $\Omega \cup \kappa \cup \hat{\Omega}$.

In Theorems 2 and 3 the reflection of solutions given in Ω, of the special system (1.1)

$$\Delta u + Eu = 0 \ , \qquad E = n \times n \text{ constant matrix} \ ,$$