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ON NORMALITY AND POINTWISE PARACOMPACTNESS

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The primary purpose of this paper is to establish some implications between normality and pointwise paracompactness in Moore spaces. In particular, it is proved that if either of two conjectures, raised by R. W. Heath and E. E. Grace, is true then each normal Moore space is indeed metrizable.

In [4], Heath and Grace raised questions regarding the substitution of normality for the condition of pointwise paracompactness in several of the theorems proved in that paper. The resulting statements appear below as Conjecture A and Conjecture B. The purpose of this note is to establish that the truth of either of the conjectures implies that each normal separable Moore space is metrizable. Thus, if either Conjecture A or Conjecture B is proved true then the condition that $2^{\aleph_0} < 2^{\aleph_1}$ would be removed from Jones' result [8, Th. 5] on the metrization of normal separable Moore spaces.

For definitions and results related to the question of metrization of normal Moore spaces, refer to [1], [2], [5], [6], [7], [8], [9], [10], [11], [13], [14].

CONJECTURE A. Suppose that S is a connected normal Moore space such that S contains no cut points and it is true that if each of P and Q is a point of S and R is a region containing P then some separable, closed connected subset N of R separates P from Q in S. Then S is separable.

CONJECTURE B. Suppose that S is a connected, locally connected, normal Moore space containing a separable closed set which separates S and each separable closed set which separates S contains two points which are separated by a separable closed set. Then S is separable.

THEOREM 1. *If Conjecture A is true then each normal separable Moore space is metrizable.*

Proof. Suppose that the theorem is false and that (S, Ω) is a normal separable nonmetrizable Moore space. There exist [8, Lemma C], in S, an uncountable set M with no limit point and a countable dense set K of $S - M$ such that each point of M is a limit point of K. The subspace $K + M$, with the relative topology, is normal, separable, nonmetrizable and a Moore space. If (S_1, Ω_1) denotes the subspace $K + M$ of (S, Ω), denote by (S_1, Ω_2) the space whose topology