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GENERALIZED ILSTOW AND FEYNMAN INTEGRALS

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Let $C[a, b]$ denote the space of continuous functions $x(t)$ defined on $[a, b] \ni x(a) = 0$. This space is called Wiener space. Using the Wiener integral we define, for each nonnegative integer M, what we call the M Ilstow, M complex Wiener, M Feynman, limiting M complex Wiener, and limiting M Feynman integrals of a functional $F(x)$ on $C[a, b]$ and show various relationships which exist between these integrals. In particular we give necessary and sufficient conditions for a finite dimensional functional $F(x)$ to be M Ilstow integrable on $C[a, b]$.

We consider the set of linear functionals $x(t_1), \cdots, x(t_n)$ where $a = t_0 < t_1 < \cdots < t_n = b$ and obtain conditions on $g_j(u) \ni$ the functional

$$F(x) = g_1[x(t_1)] \cdots g_n[x(t_n)] \tag{1.1}$$

is M Ilstow and limiting M Feynman integrable on $C[a, b]$. We then apply these results to the functional

$$F(t, \xi, x) = \exp\left(\int_a^b \theta[t - s, x(s) + \xi]ds\right)\sigma[x(t) + \xi]$$

where $0 \leqq t \leqq t_0$, $-\infty < \xi < \infty$ and $x \in C[0, t_0]$ and show that for appropriate functions $\theta(t, \xi)$ and $\sigma(\xi)$, the limiting M Feynman integral $\hat{G}(t, \xi, q)$ of $F(t, \xi, x)$ exists for

$$(t, \xi, q) \in (0, t_0) \otimes R_1 \otimes \{R_1 - \{0\}\}$$

and satisfies there the integral equation

$$\begin{aligned} \hat{G}(t, \xi, q) &= \left(\frac{-iq}{2\pi t}\right)^{1/2}\int_{-\infty}^{\infty} \sigma(u) \exp\left(\frac{qi(\xi - u)^2}{2t}\right)du \\ &\quad + \left(\frac{-iq}{2\pi}\right)^{1/2}\int_0^t (t - s)^{-1/2} ds \\ &\quad \times \int_{-\infty}^{\infty} \theta(s, u)\hat{G}(s, u, q) \exp\left(\frac{qi(\xi - u)^2}{2(t - s)}\right)du \,. \end{aligned} \tag{1.2}$$

For $M = 0$ the definitions of the above mentioned integrals reduce to the definitions of the Ilstow, complex Wiener, Feynman, limiting complex Wiener, and limiting Feynman integrals as defined by R. H. Cameron in [2]. He used the Ilstow integral as an intermediate integral in his definition of the Feynman integral. The word "Ilstow" is a contraction of "inverse Laplace Stieltjes transform of Wiener's".

Many of the theorems in this paper are generalizations of theorems in [2]. However, the techniques developed in § 4, applied when $M = 0$, allow us to reduce the hypothesis of several theorems of [2]. In particular, we obtain a condition for the Ilstow and limiting Feynman