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EXTENSIONS OF THE MAXIMAL IDEAL SPACE OF A FUNCTION ALGEBRA

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Let A be a function algebra with its maximal ideal space M_A. Let B be a function algebra such that $A \subset B \subset C(M_A)$. What can be said about M_B? We prove that $M_A = M_B$ if every point $x \in M_A$ has a fundamental neighborhood system $\{W\}$ such that the topological boundary bW of each W is contained in the Choquet boundary of A or if A is a normal function algebra. The first condition is satisfied if M_A is a one dimensional topological space. Let $H(A)$ be the function algebra on M_A generated by all functions which are locally approximable in A. We prove that $M_{H(A)} = M_A$ and then we try to generalize this result. If $f \in C(M_A)$ is such that f is locally approximable in A at every point where f is different from zero then M_A is the maximal ideal space of the function algebra generated by A and f. We also look at closed subsets F of M_A such that $M_{H(F)} = F$ where $H(F)$ is the function algebra generated by restricting to F all functions that are defined and locally approximable in A in some neighborhood of F. These sets are called natural sets. We prove that there exists a smallest natural set $B(F)$ containing a closed set F in M_A and that the Silov boundary of $H(B(F))$ is contained in F. We also find conditions that guarantee that a closed set in M_A is a natural set.

If X is a set and f is a complex-valued function defined on X then $|f|_V = \sup\{|f(x)| \mid x \in V\}$ for every $V \subset X$ and f_V is the restriction of f to V. If V is a subset of a topological space X then bV is the topological boundary of V in X. If A is a function algebra we denote by M_A its maximal ideal space, and S_A its Shilov boundary. A point $x \in M_A$ is a strong boundary point in A if $\{x\} = \cap P(f)$, where $P(f)$ are peak sets of A in M_A. We shall use the wellknown fact that S_A is the closure of the strong boundary points of A in M_A. If F is a closed set in M_A then $\mathrm{Hull}_A(F) = \{x \in M_A \mid |f(x)| \leqq |f|_F$ for every $f \in A\}$. If $x \in \mathrm{Hull}_A(F)$ we say thet F is a support of x. A minimal support of x is a support F of x such that no proper closed subset of F is a support of x. Now we have the principle of minimal supports. Let F be a minimal support of x. Suppose $\{f_n\} \in A$ is such that $|f_n|_F \leqq K$ for some constant K independent of n and $\lim |f_n|_{W \cap F} = 0$, where W is an open subset of M_A such that $W \cap F$ is not empty. Then it follows that $\lim f_n(x) = 0$. If F is a closed set in M_A then A_F is the function algebra on F generated by functions $f \in C(F)$ such that $f = g$ on F for some $g \in A$. Now M_{A_F} can be identified with