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HOMOMORPHISMS OF B*-ALGEBRAS

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This paper is divided into two sections. The first deals with Banach algebra homomorphisms of a von Neumann algebra $\mathfrak{A}$, and extends the Bade-Curtis theory for commutative B*-algebras to von Neumann algebras, as well as characterizing the separating ideal in the closure of the range of the homomorphism. The second section concerns homomorphisms of B*-algebras; the chief result being the existence of an ideal $\mathscr{I}$ with cofinite closure such that the restriction of the homomorphism to any closed, two-sided ideal contained in $\mathscr{I}$ is continuous.

1. Homomorphisms of von Neumann algebras. Let $\mathfrak{A}$ be a von Neumann algebra, and let $\nu : \mathfrak{A} \rightarrow \mathfrak{B}$ be a Banach algebra homomorphism. The reduction theory enables us to write

$$\mathfrak{A} = \sum_{i=1}^{\infty} \oplus (C(X_i) \otimes B(\mathscr{H}_i)) \oplus \mathfrak{A}_1 ,$$

where $\mathfrak{A}_1$ is the direct sum of the type II and type III parts, X_i is a hyperstonian compact Hausdorff space, and $\mathscr{H}_i$ is Hilbert space of dimension i (∞ is an allowed index of i, $\mathscr{H}_\infty$ is separable Hilbert space). It was shown in [6] that there is an integer N such that

$$\nu\Big|_{\sum_{i=N+1}^{\infty}} \oplus (C(X_i) \otimes B(\mathscr{H}_i)) \oplus \mathfrak{A}_1$$

is continuous.

Some definitions are in order.

$$S(\nu, \mathfrak{B}) = \{z \in \mathfrak{B} \mid \exists \{x_n\} \subset \mathfrak{A} \ni x_n \rightarrow 0, \quad \nu(x_n) \rightarrow z\} ;$$

$S(\nu, \mathfrak{B})$ is a closed, 2-sided ideal in $\mathfrak{B}$ ([2]). If $f \in C(X_i)$, $T \in B(\mathscr{H}_i)$, then $\langle f \otimes T \rangle$ will denote $(x, y) \in \mathfrak{A}$, where $y = 0 \in \mathfrak{A}_1$ and

$$x \in \sum_{k=1}^{\infty} \oplus (C(X_k) \otimes B(\mathscr{H}_k))$$

has $f \otimes T$ in the i^{th} component and zero in all other components. Let $\varphi_i : C(X_i) \rightarrow \mathfrak{B}$ be defined by $\varphi_i(f) = \nu(\langle f \otimes I_i \rangle)$, where I_i is the identity of $B(\mathscr{H}_i)$, and let F_i be the Bade-Curtis [1] singularity set associated with φ_i. Let $M(F_i) = \{f \in C(X_i) \mid f(F_i) = 0\}$, let $T(F_i) = \{f \in C(X_i) \mid f$ vanishes on a neighborhood of $F_i\}$, and let $R(F_i) = \{f \in C(X_i) \mid f$ is constant in a neighborhood of each point of $F_i\}$. It was shown in [6] that ν is continuous on