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# STANDARD ALGEBRAS

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**In 1948 A. A. Albert defined a standard algebra $\mathfrak{A}$ by the identities $(x, y, z) + (z, x, y) - (x, z, y) = 0$ and**

$$(x, y, wz) + (w, y, xz) + (z, y, wx) = 0 .$$

**Standard algebras include all associative algebras and commutative Jordan algebras. The radical $\mathfrak{N}$ of any finite-dimensional standard algebra $\mathfrak{A}$ is its maximal nilpotent ideal. It is known that any semisimple standard algebra is a direct sum of simple ideals, and that any simple standard algebra is either associative or a commutative Jordan algebra.**

**In this paper we study Peirce decompositions and derivations of standard algebras. We prove the Wedderburn principal theorem for standard algebras of characteristic $\neq 2$ (announced in 1950 by A. J. Penico for characteristic 0): if $\mathfrak{A}/\mathfrak{N}$ is separable, then $\mathfrak{A} = \mathfrak{S} + \mathfrak{N}$ where $\mathfrak{S}$ is a subalgebra of $\mathfrak{A}$, $\mathfrak{S} \cong \mathfrak{A}/\mathfrak{N}$. For standard algebras of characteristic 0 we prove analogues of the Malcev-Harish-Chandra theorem and the first Whitehead lemma, and we determine when the derivation algebra of $\mathfrak{A}$ is semisimple.**

Let $\mathfrak{A}$ be a nonassociative algebra over a field $F$ of characteristic $\neq 2$. In [2] Albert called $\mathfrak{A}$ a *standard algebra* in case the identities

$$(x, y, z) + (z, x, y) - (x, z, y) = 0 \tag{1}$$

and

$$(x, y, wz) + (w, y, xz) + (z, y, wx) = 0 \tag{2}$$

are satisfied, where $(x, y, z)$ denotes the associator

$$(x, y, z) = (xy)z - x(yz) .$$

Clearly every associative algebra is a standard algebra. In every nonassociative algebra one has the identity

$$(x, y, z) + (z, x, y) - (x, z, y) = [xy, z] - [x, z]y - x[y, z]$$

where $[x, y]$ denotes the commutator $[x, y] = xy - yx$. Hence (1) is equivalent to

$$[xy, z] = [x, z]y + x[y, z] , \tag{3}$$

so that every commutative algebra satisfies (1). Thus every commutative Jordan algebra of characteristic $\neq 2$ is a standard algebra.

If the characteristic is $\neq 3$, then (2) implies