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RANK k GRASSMANN PRODUCTS

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The general question concerning the structure of subspaces of a symmetry class of tensors in which every nonzero element has an irreducible representation as a sum of decomposable (or pure) elements of a given length is as yet largely unanswered. This problem relates to the problem of characterizing the linear transformations on such a symmetry class which map the set of tensors of "irreducible length" k into itself; i.e., preserves the *rank* k of the tensors. Another related problem is: "Is it possible to obtain algebraic relations involving the components of a tensor which imply it has rank ("Irreducible length") k, for any positive integer k"?

This paper is concerned mostly with the third question for the $\binom{n}{r}$-dimensional Grassmann Product Space $\wedge^r U$, where U is an n-dimensional vector space over a field F. It includes some discussion of the first question for F *algebraically closed* and $r = 2$.

A vector in $\wedge^r U$ is said to have *rank* k if it can be expressed as the sum of k, and *not less than* k, nonzero pure r-vectors in $\wedge^r U$. We denote the set of such vectors by $C_k^r(U)$. The nonzero pure products in $\wedge^r U$ have rank one.

The results obtained in this paper are as follows: (i) the rank of a vector in $\wedge^r U$ is unchanged if we extend U, (ii) in the Grassmann Algebra $\wedge^0 U + \wedge^1 U + \cdots + \wedge^r U + \cdots$, multiplication of a Grassmann product by a nonzero vector in U either annihilates it or preserves its rank, (iii) we can associate with each vector z in $C_k^r(U)$ a unique subspace $U(z)$ in U, (iv) if $z \in C_k^r(U)$ and dim $U(z)$ is rk, then z has rank k, (v) $x_1 \wedge y_1 + \cdots + x_s \wedge y_s \in C_s^2(U)$ if and only if $\{x_1, y_1, \cdots, x_s, y_s\}$ is independent. Finally, we discuss the rank two subspaces in $\wedge^2 U$ when dim $U = 4$. If F is *algebraically closed*, these subspaces are of dimension one. Otherwise, they can be different, as the examples show.

In this paper, $Q(k, t, n)$ will denote the totality of strictly increasing sequences of k integers chosen from $t, t+1, \cdots, n$; $S(k, t, n)$ the totality of sequences of k integers chosen from $t, t+1, \cdots, n$.

Let $x_1, \cdots, x_n$ be a basis of U. For $\omega = (i_1, \cdots, i_r) \in Q(r, 1, n)$, we denote the product $x_{i_1} \wedge \cdots \wedge x_{i_r}$ by $\boldsymbol{x}_\omega$.

Let p be an r-linear alternating function from $\pi_{i=1}^r E \rightarrow F$, $E = \{1, \cdots, n\}$.

We will need the following known result.

THEOREM 1. (See [2], p. 289-312.) *Let*