PACIFIC JOURNAL OF MATHEMATICS
Vol. 29, No. 2, 1969

A GREEN'S FUNCTION APPROACH TO PERTURBATIONS OF PERIODIC SOLUTIONS

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Consider the nonlinear differential system

$$\dot{y} = A(t) \cdot y + \varepsilon q(t, y, \varepsilon) \tag{1.1}$$

where y, q are column n-vectors, q is continuous in (t, y, ε) and has continuous second partial derivatives with respect to y, ε for all values of t, $0 \leqq y \leqq R$ for some $R > 0$ and $0 \leqq \varepsilon \leqq \varepsilon_0$ for some $\varepsilon_0 > 0$. Further assume $A(t)$ is an $n \times n$ matrix such that $A \in C^1$, and both A and q are periodic in t with period T. Associated with system (1.1) are the general homogeneous and nonhomogeneous equations

$$\dot{y} = A(t) \cdot y \tag{1.2}$$

$$\dot{y} = A(t) \cdot y + f(t) \tag{1.3}$$

where $f(t)$ is an arbitrary n-vector function periodic in t of period T. In this paper we consider the classical problem of proving the existence of T-periodic solutions $y = y(t)$ of (1.1) when the homogeneous system (1.2) has nontrivial T-periodic solutions.

In his book "Oscillations in Nonlinear Systems" [5], J. K. Hale treats this problem in Chapter 11 as an extension of the general method developed in Chapter 6 for the case when $A(t)$ is a constant matrix A. Among other treatments of this problem is the work of D. C. Lewis [6] utilizing a generalized Green's matrix. It is the aim of the present paper to incorporate certain ideas from [6] into the general method employed in [5] in Chapter 6. We feel that the approach presented here brings into sharper focus the generality of the methods employed in [5] and also has certain computational advantages which will be mentioned in the body of the paper. The present paper has many points of contact with a recent paper of S. Bancroft, J. K. Hale, and D. Sweet [2]. See also [1], [7] and [8].

2. We must first consider the general nonhomogeneous equation (1.3). We will state a necessary and sufficient condition for the existence of T-periodic solutions of (1.3) when (1.2) has nontrivial T-periodic solutions. Following [6] we then construct a generalized Green's function in order to find an explicit periodic solution of (1.3).

Let $S = \{f : f(t) = f(t + T); f \in C;$ and $f(t)$ is a real valued n-vector for all $t\}$. Then S is a Banach space if we define the norm $V(f)$ to