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EIGENVALUES OF THE ADJACENCY MATRIX OF CUBIC LATTICE GRAPHS

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A cubic lattice graph is defined to be a graph G, whose vertices are the ordered triplets on n symbols, such that two vertices are adjacent if and only if they have two coordinates in common. If $n_2(x)$ denotes the number of vertices y, which are at distance 2 from x and $A(G)$ denotes the adjacency matrix of G, then G has the following properties: (P_1) the number of vertices is n^3. (P_2) G is connected and regular. (P_3) $n_2(x) = 3(n-1)^2$. (P_4) the distinct eigenvalues of $A(G)$ are -3, $n-3$, $2n-3$, $3(n-1)$. It is shown here that if $n > 7$, any graph G (with no loops and multiple edges) having the properties $(P_1)-(P_4)$ must be a cubic lattice graph. An alternative characterization of cubic lattice graphs has been given by the author (J. Comb. Theory, Vol. 3, No. 4, December 1967, 386-401).

We shall consider only finite undirected graphs without loops or multiple edges. A cubic lattice graph with characteristic n is defined to be a graph whose vertices are identified with the n^3 ordered triplets on n symbols, with two vertices adjacent if and only if their corresponding triplets have two coordinates in common. If $d(x, y)$ denotes the distance between two vertices x and y and $\Delta(x, y)$ the number of vertices adjacent to both x and y, then it has been shown by the author [6] that for $n > 7$, the following properties characterize the cubic lattice graph with characteristic n:

(b_1) The number of vertices is n^3.

(b_2) The graph is connected and regular of degree $3(n-1)$.

(b_3) If $d(x, y) = 1$, then $\Delta(x, y) = n - 2$.

(b_4) If $d(x, y) = 2$, then $\Delta(x, y) = 2$.

(b_5) If $d(x, y) = 2$, there exist exactly $n - 1$ vertices z, adjacent to x such that $d(y, z) = 3$.

Dowling [4] in a note has shown that the property (b_5) is implied by properties $(b_1) - (b_4)$ for $n > 7$. Hence for $n > 7$, $(b_1) - (b_4)$ characterize a cubic lattice graph with characteristic n.

The adjacency matrix $A(G)$ of a graph G is a square $(0, 1)$ matrix whose rows and columns correspond to the vertices of G, and $a_{ij} = 1$ if and only if i and j are adjacent. Let $n_2(x)$ denote the number of vertices y at distance 2 from x.

A cubic lattice graph G with characteristic n has the following properties:

(P_1) The number of vertices is n^3.