PACIFIC JOURNAL OF MATHEMATICS
Vol. 30, No. 1, 1969

ON THE TENSOR PRODUCTS OF VON NEUMANN ALGEBRAS

JUN TOMIYAMA

Let A and B be C^*-algebras and let $A \otimes_\alpha B$ be their C^*-tensor product with Turumaru's α-norm. The author has previously defined mappings $R_\varphi: A \otimes_\alpha B \rightarrow B$ and $L_\psi: A \otimes_\alpha B \rightarrow A$ via bounded linear functionals φ on A and ψ on B, as follows:

$$R_\varphi\Big(\sum_{i=1}^{n} a_i \otimes b_i\Big) = \sum_{i=1}^{n} < a_i, \varphi > b_i \ ,$$

$$L_\psi\Big(\sum_{i=1}^{n} a_i \otimes b_i\Big) = \sum_{i=1}^{n} < b_i, \psi > a_i \ ,$$

and has shown how the families $\{R_\varphi \mid \varphi \in A^*\}$ and $\{L_\psi \mid \psi \in B^*\}$ determine the structure of the tensor product of A and B. Moreover, in a joint paper with J. Hakeda the author also proved the existence of these kinds of mappings in tensor products of von Neumann algebras and gave some of their applications. Further applications of these mappings are shown in the present paper.

Theorem 2 says that the product $M \otimes N$ has property L if one of the factors M or N has property L. This answers a question of Sakai. It can be shown that the above families of mappings determine completely the tensor products of von Neumann algebras (Theorem 3). Theorem 4 shows that if π_1 and π_2 are projection of norm one from M_1 and N_1 to their subalgebras M_2 and N_2, then there exists, without assuming their σ-weak continuity, a projection of norm one π from $M_1 \otimes \mathscr{B}(K) \cap \mathscr{B}(H) \otimes N_1$ to $M_2 \otimes \mathscr{B}(K) \cap \mathscr{B}(H) \otimes N_2$ such that $\pi(a \otimes b) = \pi_1(a) \otimes \pi_2(b)$, where $a \in M_1$ and $b \in N_1$.

We always denote by $M \otimes N$ the tensor product of the von Neumann algebras M and N and by $M \otimes_\alpha N$ their tensor product as C^*-algebras. M^* means the conjugate space of M and M_* the predual of the von Neumann algebra M.

The following theorem is the basic result cited in the above introduction; it is a more precise version of Lemma 2.5 of [1]. We give the proof for the sake of completeness.

THEOREM 1. *Let M and N be von Neumann algebras and $M \otimes N$ their tensor product. Then for each $\varphi \in M_*$* (resp. $\psi \in N_*$) *there exists a σ-weakly continuous mapping $R_\varphi: M \otimes N \longrightarrow N$* (resp. *$L_\psi: M \otimes N \longrightarrow M$*) *satisfying the following conditions:*