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COVERING GROUPS OF GROUPS OF LIE TYPE

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A construction for a central extension of a group satisfying a certain set of axioms has been given by C. W. Curtis. These groups are called groups of Lie type. The construction is based on that given by R. Steinberg for covering groups of the Chevalley groups. The central extensions constructed by Curtis, however, are not covering groups in the sense of being universal central extensions, as he shows by an example. Here, the Steinberg construction is considered for a more restricted class of groups of Lie type. It is shown that in this case, the central extension is a covering. It is also shown that this more restricted definition of groups of Lie type still includes the Chevalley and twisted groups, with certain exceptions.

To fix our terminology: a universal central extension is one which factors through any other central extension. A covering is a universal central extension, no subgroup of which is also an extension of the same group. $(x, y) = xyx^{-1}y^{-1}$, $a^b = bab^{-1}$, and (G, G) is the commutator subgroup of G. G is perfect if $G = (G, G)$. $L_n(K)$ denotes the Chevalley group of type L and rank n over the field K. Twisted groups are defined here to be the (algebraic) nonnormal forms as constructed by D. Hertzig [5, 6], R. Steinberg [9] and J. Tits [14, 15]. (Hertzig also shows that the Chevalley and twisted groups include all finite simple algebraic groups.)

The Chevalley groups are simple [1, 10, 16], hence perfect. This means that a perfect covering group exists, and the covering group and its factor maps are unique. Steinberg shows that the covering can be constructed as the abstract group given by suitably chosen generators and relations from the Chevalley group [11]. Exceptions occur: $A_1(2)$, $A_1(3)$, $B_2(2)$ and $G_2(2)$ are not simple, and Steinberg's construction doesn't work when $|K| = 2$, 3, or 4, or $G = A_1(9)$.

The construction of the covering group Δ easily extends to a group G with Bruhat decomposition as defined by Curtis [2]. However, in this case Δ need not be the covering group of G. By placing additional conditions on G, Curtis shows that Δ has a Bruhat decomposition, with the same (isomorphic) Weyl group as G, is a central extension, and is "almost" universal [3, Ths. 1.4, 1.7].

The next section gives the set of axioms which characterize the class of groups of Lie type. Our main theorem (6.2) can be stated as: