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INTEGRABILITY OF ALMOST COSYMPLECTIC STRUCTURES

SAMUEL I. GOLDBERG AND KENTARO YANO

Integrability conditions for almost cosymplectic structures on almost contact manifolds are obtained. Examples of these structures are given by taking the direct product of an almost Kaehler manifold with a line R or a circle S^1. If the curvature transformation of the metric associated with an almost cosymplectic space M commutes with the fundamental singular collineation ϕ of M, then the related almost contact structure on M gives rise to a complex structure on $M \times R$. The manifold M is then a cosymplectic space, examples being given by taking the direct product of a Kaehler manifold with R or S^1. In particular, an almost cosymplectic manifold is cosymplectic if and only if it is locally flat.

In a recent paper [3] one of the authors examined the integrability of almost Kaehler manifolds $M(J, g)$ showing, in particular that if the curvature transformation of the almost Kaehler metric g commutes with the almost complex structure tensor J, then J is integrable, that is, the structure (J, g) on M is Kaehlerian. This is also a special case of a theorem due to A. Gray [4] whose methods apparently do not extend to include the results of this paper which, therefore, complement those given by him. It was also proved that an almost Kaehler space of constant curvature is a Kaehler space if and only if it is locally fiat. Our main purpose here is to extend these results to almost cosymplectic manifolds.

THEOREM 1. *If the curvature transformation of the metric g of the almost cosymplectic manifold $M(\phi, \eta, g)$ commutes with ϕ, then M is normal, that is, it is a cosymplectic manifold.*

A cosymplectic manifold of constant curvature is locally flat in the given metric [1]. For almost cosymplectic spaces we have

COROLLARY 1.1. *An almost cosymplectic manifold of constant curvature is cosymplectic if and only if it is locally flat.*

By imposing a condition on the scalar curvature of an almost cosymplectic space, the same conclusion prevails. Examining the Nijenhuis torsion of the collineation ϕ, we find that a 3-dimensional almost cosymplectic manifold is cosymplectic if its fundamental vector field is a Killing field.