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LINEAR EIGENVALUES AND A NONLINEAR BOUNDARY VALUE PROBLEM

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In this paper a nonlinear boundary value problem for elliptic partial differential equations is considered. The principal result generalizes a previous result on a two point boundary value problem for a nonlinear second order ordinary differential equation. The solvability condition obtained for the nonlinear problem is related to the eigenvalues of an associated linear problem.

In [5] the second author and D. E. Leach considered the two point boundary value problem

$$\begin{aligned} &u'' + p(x, u, u')u = h(x, u, u') \\ &u(0) = a, \ u(\pi) = b \ . \end{aligned} \tag{1.1}$$

It was shown that, if for some integer N there exist numbers γ_N and γ_{N+1} such that

$$N^2 < \gamma_N \leqq p(x, s, r) \leqq \gamma_{N+1} < (N+1)^2 \ ,$$

if $p(x, s, r)$ and $h(x, s, r)$ are continuous on $[0, \pi] \times (-\infty, \infty) \times (-\infty, \infty)$, and h is bounded, then the problem (1.1) has at least one solution.

In this paper we consider the n-dimensional analogue of the problem (1.1) which is

$$\begin{aligned} &\Delta u + p\left(x, u, \frac{\partial u}{\partial x_1}, \cdots, \frac{\partial u}{\partial x_n}\right)u = h\left(x, u, \frac{\partial u}{\partial x_1}, \cdots, \frac{\partial u}{\partial x_n}\right) , \\ &u(x) = g(x) \text{ on } \partial D \ , \end{aligned} \tag{1.2}$$

where D is a domain in R^n and Δ is the n-dimensional Laplacian. The corresponding result is that if D is a Dirichlet domain, if there exist numbers γ_N and γ_{N+1} such that

$$\alpha_N < \gamma_N \leqq p(x, t, s_1, \cdots, s_n) \leqq \gamma_{N+1} < \alpha_{N+1} \ ,$$

for $(x, t, s_1, \cdots, s_n) \in \bar{D} \times R^{n+1}$ where

$$\alpha_1 \leqq \alpha_2 \leqq \cdots \leqq \alpha_K \leqq \alpha_{K+1} \leqq \cdots \ ,$$

are the eigenvalues of the problem

$$\Delta u + \alpha u = 0, \qquad u = 0 \text{ on } \partial D \ , \tag{1.3}$$

if $p(x, t, s_1, \cdots, s_n)$, $h(x, t, s_1, \cdots, s_n)$ are continuous and h bounded on