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ABSOLUTE SUMMABILITY BY RIESZ MEANS

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In this paper we ensure the absolute Riesz summability of Lebesgue-Fourier series under more liberal conditions imposed upon the generating function of Lebesgue-Fourier series and by taking more general type of Riesz means than whatever the present author has previously taken in proving the corresponding result. Also we give a refinement over the criterion previously proved by author himself.

1. **Definitions and notations.** Let $\sum_{n=0}^{\infty} a_n$ be a given infinite series with the sequence of partial sums $\{s_n\}$. Throughout the paper we suppose that

$$\lambda_n = \mu_0 + \mu_1 + \mu_2 + \cdots + \mu_n \longrightarrow \infty, \text{ as } n \longrightarrow \infty . \tag{1.1}$$

The sequence-to-sequence transformation

$$t_n = \frac{1}{\lambda_n}\sum_{\nu=0}^{n} \mu_\nu s_\nu , \tag{1.2}$$

defines the Riesz means of sequence $\{s_n\}$ (or the series $\sum_{n=0}^{\infty} a_n$) of the type $\{\lambda_{n-1}\}$ and order unity.[1] If $t_n \to s$, as $n \to \infty$, the sequence $\{s_n\}$ is said to be summable $(R, \lambda_{n-1}, 1)$ to the sum s and if, in addition, $\{t_n\} \in BV$,[2] then it is said to be absolutely summable $(R, \lambda_{n-1}, 1)$, or summable $|R, \lambda_{n-1}, 1|$ and symbolically we write $\sum_{n=0}^{\infty} a_n \in |R, \lambda_{n-1}, 1|$.

The series $\sum_{n=1}^{\infty} a_n \in |R, \lambda_{n-1}, 1|$, if

$$\sum_{n=0}^{\infty} \left| \frac{\Delta\lambda_n}{\lambda_n\lambda_{n+1}} \sum_{\nu=0}^{n} \lambda_\nu a_{\nu+1} \right| < \infty .$$

Let $f(t)$ be a periodic function with period 2π and integrable in the sense of Lebesgue over $(-\pi, \pi)$. Without any loss of generality the constant term of the Lebesgue-Fourier series of $f(t)$ can be taken to be zero, so that

$$\int_{-\pi}^{\pi} f(t)dt = 0 ,$$

and

$$f(t) \sim \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} A_n(t) .$$

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[1] It is some-times called $(\bar{N}, \mu_n)$ mean, or (R, μ_n) mean, or Riesz's discrete mean of 'type' λ_{n-1} and 'order' unity and is, in fact, equivalent to the usually known $(R, \lambda_{n-1}, 1)$ mean. An explicit proof of it is contained in Iyer [**6**]. Also see Dikshit [**3**].

[2] '$\{t_n\} \in BV$' means $\sum_n |\Delta t_n| < \infty$, when $\Delta t_n = t_n - t_{n+1}$.