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SETS WHICH CAN BE MISSED BY SIDE APPROXIMATIONS TO SPHERES

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In this paper it is proved that a 2-sphere S in E^3 can be side approximated from Int S so as to miss a closed subset X of S if and only if $(S \cup \mathrm{Int}\, S) - X$ is 1-ULC.

If $X = \{p\}$ is a singleton, then the 1-ULC property described in the first paragraph reduces to a property studied by McMillan [22] in connection with piercing points p of a crumpled cube (see (0.2) below). Our work was motivated by his paper although our methods bear little relationship to his.

Our result has as immediate corollaries a number of important, well-known theorems, four of which we now mention:

(0.1) ([2. Th. 2]). A 2-sphere S in E^3 is tame if $E^3 - S$ is 1-ULC.

(0.2) ([22, Th. 1]). If C is a crumpled cube in E^3 and $p \in \mathrm{Bd}\, C$, then p is a piercing point of C if and only if $C - p$ is 1-ULC.

(0.3) ([19, Th. 3]). If S and S' are 2-spheres in E^3 and $X \subset U \subset S \cap S'$, where X is compact and U is open in S and S'. Then $(*, X, S')$ is satisfied if and only if $(*, X, S)$ is satisfied.

(0.4) ([8] and [10]). If S is a 2-sphere in E^3, X is a compact subset of S, and $X = \bigcup_{i=1}^{\infty} X_i$, where each X_i is compact and satisfies $(*, X_i, S)$, then $(*, X, S)$ is satisfied.

Properties similar to the 1-ULC property described in the first paragraph have been studied previously (e.g., properties (A, F, S) and (B, F, S) in [19]). However, the properties studied have, in general, had only restricted application to the question of determining which subsets of a 2-sphere can be missed by side approximations to that sphere and thus have been inapplicable in results such as (0.2), (0.3), and (0.4).

Precise definitions appear in § 1. Our main result, mentioned in the first paragraph, is proved in § 2. We also present in § 2 a slightly strengthened version of our main result for use in another paper (Lemma 2.5 and Theorem 2.6). The four corollaries mentioned above are proved in § 3. In § 3 we also remark on ways in which our main result can be used to sharpen other results which have appeared in the literature.

The part of this paper actually necessary for the proof of Theorem (0.1) appears in five short paragraphs. This proof is considerably shorter and conceptually easier to follow than the original proof. Our proof of Theorem (0.3) is likewise much shorter than the original. The