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COMPACT, DISTRIBUTIVE LATTICES OF FINITE BREADTH

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Necessary and sufficient conditions are given for a complete distributive lattice with topology to be embeddable in the product of n complete chains, where the embedding is required to be simultaneously algebraic and topological. Corollaries are (i) a characterization of those compact topological lattices which can be embedded in an n-cell and (ii) the fact that breadth provides a bound on the topological dimension for compact, distributive topological lattices of finite breadth.

These corollaries prove conjectures of Anderson [2] and Dyer and Shields [7], respectively.

In considering topologies on a lattice, there are two possible approaches. The "extrinsic" point of view presupposes a *given* Hausdorff topology for which the lattice operations are continuous, much as in the study of topological groups (cf. [2], [7]). Such a structure is commonly called a *topological lattice*, and a typical problem is to relate its topological dimension to the algebraic lattice structure.

The "intrinsic" point of view, on the other hand, is to examine topologies on a lattice which arise naturally from the lattice structure. Prominent examples are the Frink and Birkhoff interval topologies and the topology generated by order convergence [4, Ch. 10]. Such topologies usually fail to give a topological lattice in the above sense, even for complete distributive lattices. A typical problem is to study conditions under which an intrinsic topology does give a topological lattice.

There is one important class of lattices for which both points of view merge, namely, those compact topological lattices which are known to be embeddable, topologically and algebraically, in a product of complete chains. Specifically, if L is such a lattice, all the commonly used intrinsic topologies on L coincide and constitute the only compact Hausdorff topology under which L is a topological lattice.

In this connection, the following complementary facts are known. Let L be a distributive topological lattice with compact topology. By an "algebraic" embedding of L we mean an embedding which preserves the binary lattice operations.

1. If L has infinite breadth, it may not be possible to embed L topologically and algebraically in any product of chains [12].

2. If L has finite breadth n, then L can be embedded topolo-