PACIFIC JOURNAL OF MATHEMATICS
Vol. 35, No. 2, 1970

QUASIFIBRATION AND ADJUNCTION

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This paper is concerned with the preservation of quasifibration under pair adjunction of Hurewicz fibrations, and the preservation of weak homotopy equivalence under pair adjunction of identity maps.

The foundations of the theory of quasifibrations were laid by Dold and Thom in their paper [3] in which they proved that the homotopy groups of the infinite symmetric product of a space X were naturally isomorphic with the integral homology groups of X. Another application was soon given by Dold and Lashof [2] generalizing Milnor's construction [9] of a universal principal fibre bundle with given structure group and their results were further generalized by Stasheff [13], Milgram [8], Steenrod [15] and Stasheff [14]. Other applications of quasifibrations occur in [6], [5] and [1]. Since, as a generalisation of fibration, quasifibration has a serious deficiency (it is not preserved under pull-back) one may well ask why it has proved to be so useful. A study of the papers referred to reveals that it is essentially the behaviour of quasifibration with respect to adjunction which is involved. However, the relevant arguments mostly rely on a basic lemma of [3] (lemma 2.10) and proceed ad hoc.

Let $p: P \to P'$, $t: T \to T'$, $q: Q \to Q'$ be *pairs* (i.e., continuous maps) and let $\phi = (f, f'): p \to t$, $\gamma = (g, g'): p \to q$ be pair maps and consider the push-out diagram

$$\begin{tikzcd} p \arrow[r, "\gamma"] \arrow[d, "\phi"'] & q \arrow[d, dashed] \\ t \arrow[r, dashed] & r \end{tikzcd} \tag{0.1}$$

in the category of pairs. γ is a *weak homotopy equivalence of fibres* (WHEF) if, for each $x \in P'$, the induced map

$$g'': p^{-1}(x) \longrightarrow q^{-1}(g'x)$$

is a weak homotopy equivalence. We shall prove the following theorem.

THEOREM 0.2. *If f' is a closed cofibration, if t is a fibration, if p is the pull-back of t over f', if q is a quasifibration and if γ is a* WHEF *then r is a quasifibration.*