PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 2, 1971

ON WEIGHTED POLYNOMIAL APPROXIMATION OF ENTIRE FUNCTIONS

B. A. TAYLOR

An existence theorem for the $\bar{\partial}$ operator is used here to prove some results on weighted approximation of entire functions. Theorem 2 shows that if ϕ is a convex function on $\mathfrak{C}^n = R^{2n}$ such that the Hilbert space of all entire functions f with $\int |f|^2 e^{-\phi} d\lambda < +\infty$ ($d\lambda$ Lebesgue measure) contains the polynomials, then the polynomials are dense in this Hilbert space. Two approximation theorems are also given which are related to the theory of quasi-analytic functions.

The method used here is the analogue for $\mathfrak{C}^n$ of the method used by Hörmander in [7] (see also [6]) to prove approximation theorems for analytic functions in domains of holomorphy. We apply an existence theorem for the $\bar{\partial}$ operator, Theorem 4.4.1 of [7], to prove our Theorem 1, which gives essentially a modification of the results proved in Lemma 4.3.1 and Theorem 4.4.4 of [7]. Our proof is somewhat simpler than the corresponding proofs in [7] because we are working on $\mathfrak{C}^n$ rather than an arbitrary domain of holomorphy, which makes several technical details easier. The rest of the paper then deals with applications of Theorem 1 to weighted approximation of entire functions.

We point out that most of the results proved in this paper can be proved by other methods. For example, Theorems 2 and 5 can be deduced from results in [3]. However, the theorems in this paper are much simpler than the corresponding results of [3]. The methods used here also demonstrate that Hörmander's L^2 estimates for the $\bar{\partial}$ operator are non-trivial even in one variable, as has already been pointed out by Kiselman [10].

1. **Application of the $\bar{\partial}$ existence theorem.** We recall briefly some of the results of Hörmander as presented in [7]. Throughout the following ϕ denotes a plurisubharmonic function on $\mathfrak{C}^n = \mathfrak{C} x \cdots x \mathfrak{C}$ (n times, $\mathfrak{C}$ = complex numbers), and $L^2(\phi)(=L^2(\mathfrak{C}^n, \phi))$ denotes the Hilbert space of functions on $\mathfrak{C}^n$ which are square integrable with respect to the measure $e^{-\phi} d\lambda$, where $d\lambda$ is the Lebesgue measure. Similarly, $L^2_{p,q}(\phi)$ is the space of differential forms of type (p, q) with coefficients from $L^2(\phi)$. The collection of all entire functions $f \in L^2(\phi)$ is denoted $\mathfrak{A}(\phi)$. A function $f \in L^2(\phi)$ is n $\mathfrak{A}(\phi)$ if and only if $\bar{\partial} f = \sum_{j=1}^n (\partial f / \partial \bar{z}_j) d\bar{z}_j$ is the zero $(0, 1)$ form (with derivatives taken in the sense of distributions).

We shall use a special case of Theorem 4.4.1 of [7].