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A NOTE ON ANNIHILATOR IDEALS OF COMPLEX BORDISM CLASSES

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Recent studies of the complex bordism homology theory $\Omega_*^U(\ )$ have shown that for a finite complex X the integer hom-dim $_{\Omega_*^U}\Omega_*^U(X)$ provides a useful numerical invariant measuring certain types of complexity in X. Associated to an element $\alpha \in \Omega_*^U(X)$ one has the annihilator ideal $A(\alpha) \subset \Omega_*^U$. Numerous relations between $A(\alpha)$ and hom-dim $_{\Omega_*^U}\Omega_*^U(X)$ are known. In attempting to deal with these invariants it is of course useful to study special cases, and families of special cases. In this note we study the annihilator ideal of the canonical element $\sigma \in {}_{2N}^{U}\Omega(X)$ where X is a complex of the form

$$S^{2N} \cup_p e^{2N+1} \cup e^{2N+2n_1-1} \cup \cdots \cup e^{2N+2n_k-1}$$

and $N >> n_1, \cdots n_k > 1$, and p an odd prime. We show that $A(\sigma) \not\ni [V^{2p^2-2}], \cdots, [V^{2p^s-2}], \cdots,$ where $[V^{2p^s-2}] \in \Omega_{2p^s-2}^U$ is a Milnor manifold for the prime p. This provides another piece of evidence that for such a complex X, hom-dim $_{\Omega_*^U}\Omega_*^U(X)$ is 1 or 2.

In [9], [11] and [12] the study of the annihilator ideal of the canonical class $\alpha \in \widetilde{\Omega}_0^U(X)$ in a stable complex X of the form

$$X = S^0 \cup_p e^1 \cup_f e^{2n-1}$$

played a crucial role in the applications of [9] and [11] to the stable homotopy of spheres. In the closing remarks of [4] it was suggested that more generally for a stable complex of the form (where p is an *odd* prime)

$$Y = S^0 \cup_p e^1 \cup_{f_1} e^{2n_1-1} \cdots \cup_{f_k} e^{2n_k-1}$$

the annihilator ideal of the canonical class $\alpha \in \widetilde{\Omega}_0^U(Y)$ had the form $(p, [CP(p\text{-}1)]^t)$ for some integer t, and that hom-dim $_{\Omega_*^U}\Omega_*^U(Y) \leqq 2$. Our objective in this note is to make the following elementary contribution to these matters.

THEOREM. *Let Y be a stable complex of the form*

$$S^0 \cup_p e^1 \cup_{f_1} e^{2n_1-1} \cup_{f_2} e^{2n_2-1} \cdots \cup_{f_k} e^{2n_k-1}$$

where p is odd prime. Let $\sigma \in \widetilde{\Omega}_0^U(Y)$ denote the canonical class. Then