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AN ALGEBRAIC PROPERTY OF THE TOTALLY SYMMETRIC LOOPS ASSOCIATED WITH KIRKMAN-STEINER TRIPLE SYSTEMS

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The concept of an x-root of degree r in a loop of order n is introduced. It is shown that the totally symmetric loop of order $n+1$ derived from any Kirkman-Steiner triple system of order n admits a maximal identity-root. A statistical-combinatorial application of this algebraic property is then indicated. Finally, two open problems are also given.

A mathematical system consisting of an n-set Ω and a binary operation $*$ is said to form a loop of order n if the following axioms are satisfied:

(1) Ω contains an identity element e such that $x*e = e*x = x$ for every x in Ω.

(2) Any two of the elements in the equation $x*y = z$ uniquely determine the third.

Since the notation $x*y$ is too bulky we shall use, hereafter, the notation xy instead. A loop is said to be a totally symmetric loop if it also satisfies

(3) $xy = yx$ and $x(xy) = y$ for all x and y in Ω.

In this paper, we shall introduce and study an algebraic property of totally symmetric loops of order $n \equiv 3 \pmod 6$. In the final part of this paper we shall indicate, briefly, a statistical-combinatorial application of this study. A few open questions are also stated.

We begin by introducing and reviewing certain concepts and results that will be relevant to our forthcoming results.

DEFINITION 1. We say a loop $\mathscr{L}$ of order n accepts a

$$(k_1, k_2, \cdots, k_r)$$

orthogonal partition if the n^2 cells in the Cayley table of $\mathscr{L}$ can be divided into r mutually disjoint exhaustive sets $S_1, S_2, \cdots, S_r$; in such a way that (1) S_i has k_i cells from each row and each column, (2) each element of $\mathscr{L}$ appears k_i times in the cells of S_i,