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AN ELEMENTARY DEFINITION OF SURFACE AREA IN E^{n+1} FOR SMOOTH SURFACES

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The present paper concerns the difficulty which one encounters in text books of Advanced Calculus of giving a simple and elementary definition of area of a smooth non-parametric surface in E^{n+1} such that, within the same elmentary framework, one can then prove that the area so defined is equal to the classical area integral.

The authors were first made aware of the considerable interest of such a task in 1955 with the publication of Angus Taylor's now classic textbook "Advanced Calculus". The following statement is taken from page 384 of this book:

"It is logically and aesthetically desirable to have a definition of surface area which is directly geometric, and which does not put too many restrictions on the surface. A good definition ought not to depend upon the method of representing the surface analytically, and should not be limited to smooth surfaces. The demand for such a definition poses a very difficult problem, however. It may surprise the student to know that the problem has occupied the attention of many able mathematicians over the last fifty years, and that the end of research on the question is not yet in sight."

In the present paper we present an idea which seems to answer the questions raised by Angus Taylor for surfaces S: $z=f(x_1, \cdots, x_n)$, which are continuous with their first order partial derivatives. The idea is to develop a scheme for the construction of sequences of suitably chosen polyhedra inscribed within the given surface, such that the corresponding sequences of the polyhedral areas converge to the classical area integral for the surface, and hence to the Lebesgue area of S.

In previous papers [1], [7] we discussed our definition of area for surfaces S: $z=f(x_1, x_2)$. In [7] we took in consideration surfaces $z=f(x_1, x_2)$ with f continuous with its first order partial derivatives. In [1] we gave a necessary and sufficient condition in order that for a surface $z=f(x_1, x_2)$ there are sequences of inscribed polyhedra satisfying the requirements of our definitions (see [1]).

J. A. Serret [6] in 1868 proposed a geometric definition of area, but H. A. Schwartz [5] in 1882 proved that Serret's definition was incorrect. Other geometric definitions of area and constructions have been proposed, and we mention here for example the ones of S. Kempisty [3] for surfaces S: $z=f(x_1, x_2)$ with f absolutely continuous in the sense of Tonelli. For general expositions concerning area, in particular, Le-