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THE HASSE-WITT-MATRIX OF SPECIAL PROJECTIVE VARIETIES

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The Hasse-Witt-matrix of a projective hypersurface defined over a perfect field k of characteristic p is studied using an explicit description of the Cartier-operator. We get the following applications. If L is a linear variety of dimension $n+1$ and X a generic hypersurface of degree d, which divides $p-1$, then the Frobenius-operator $\mathscr{F}$ on $H^n(X \cdot L; \mathscr{O}_{L \cdot Y})$ is invertible.

As another application we prove the invertibility of the Hasse-Witt-matrix for the generic curve of genus two. We don't study the Frobenius $\mathscr{F}$ directly, but the Cartier-operator [1]. It is well-known, that for curves Frobenius and Cartier-operator are dual to each other under the duality of the Riemann-Roch theorem. A similar fact is true for higher dimension via Serre duality. We have therefore to extend to the whole "De Rham" ring the description of the Cartier-operator given in [4] for 1-forms. We give this extention in §1. Diagonal hypersurfaces are studied in §2 and the invertibility of the Hasse-Witt-matrix is proved, if the degree divides $p-1$. The same theorem for the generic hypersurface follows then from the semicontinuity of the matrix rank. The §3 is devoted to hyperelliptic curves and is intended as a preparation for a detailed study of curves of genus two.

1. The Cartier-operator of a projective hypersurface. We extend the explicit construction of the Cartier-operator given in [4] to the whole "De Rham" ring, but restrict ourself to projective hypersurfaces.

As an application we show: Let V be a projective hypersurface of dimension $n-1$, defined by a diagonal equation $F(X) = \sum_{i=0}^{n} a_i X_i^r$, $a_i \in k$ a perfect field of $\operatorname{char} k = p > 0$, $a_i \neq 0$. Let X be a linear variety of dimension $t+1$. If r divides $p-1$, then

$$\mathscr{F}: H^t(X \cdot V, \mathscr{O}_{X \cdot V}) \to H^t(X \cdot V, \mathscr{O}_{X \cdot V})$$

is invertible, $\mathscr{F}$ being the induced Frobenius endomorphism. We have to rely on a technical proposition, which is a collection of some lemmas in [4]. We give first the proposition.

PROPOSITION 1. *Let*

$$\psi: k[T] \to k[T] \qquad (T = (T_1, \cdots, T_n))$$