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EXISTENCE OF SPECIAL K-SETS IN CERTAIN LOCALLY COMPACT ABELIAN GROUPS

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In all that follows, G is an infinite, nondiscrete, locally compact T_0 abelian group with character group X and Δ is a nonempty subset of X. In a standard proof of the existence of infinite (in fact, perfect) Helson sets (see for example Hewitt and Ross) it is shown that each nonvoid open subset of an arbitrary G contains a K-set (terminology of Hewitt and Ross) homeomorphic to Cantor's ternary set (or, in the terminology of Rudin, a Kronecker set or a set of type K_a homeomorphic to the Cantor set). In this paper, it is shown that $K_{0,\Delta}$-sets or $K_{a,\Delta}$-sets homeomorphic to the Cantor set exist in profusion in a large class of infinite nondiscrete locally compact T_0 abelian groups G, provided that $\bar{\Delta}$ is not compact. (A nonvoid subset E of G is called a $K_{0,\Delta}$-set if for every continuous function from E to $\mathbf{T}$, the circle group, and every $\varepsilon > 0$, there is a $\gamma \in \Delta$ such that $|\gamma(x) - f(x)| < \varepsilon$ for all $x \in E$. Let a be an integer greater than one. A nonvoid subset E of G is called a $K_{a,\Delta}$-set if it is totally disconnected and every continuous function on E with values in the set of ath roots of unity is the restriction to E of some $\gamma \in \Delta$.)

The following theorems will be proved.

THEOREM I. *Let G be compact. Let Δ be infinite. Suppose that, except for the character which is identically* 1, *$\Delta\Delta^{-1}$ consists solely of elements of infinite order.* (*This condition is satisfied automatically if G is connected, for then X is torsion-free.*) *Then every nonvoid open set in G contains a $K_{0,\Delta}$-set homeomorphic to the Cantor set.*

THEOREM II. *Let G be locally connected. Suppose that $\bar{\Delta}$ is not compact. Then every nonvoid open set in G contains a $K_{0,\Delta}$-set homeomorphic to the Cantor set.*

THEOREM III. *Let G be a compact torsion group. Let Δ be infinite. Then there is an integer $a \geqq 2$ such that every nonvoid open set in G contains a translate of a $K_{a,\Delta}$-set homeomorphic to the Cantor set.*

1. Preliminaries.

NOTATION 1.1. We denote Haar measure on G by m, with $m(G) = 1$ when G is compact. When H is a subgroup of G, we write