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TOPOLOGICAL GROUPS WHOSE UNDERLYING SPACES ARE SEPARABLE FRÉCHET MANIFOLDS

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Let G be a topological group and let G^* denote the space of all Lebesgue measurable functions from the unit interval $[0, 1]$ into G with the topology of convergence in measure. With this topology and with pointwise multiplication as the group operation, G^* is a topological group. If G is separable and has a complete metric and has more than one point, then Bessaga and Pełczyński have shown that G^* is homeomorphic to l_2, separable infinite-dimensional Hilbert space. This fact is used in this paper to show the existence of separable Fréchet manifolds which are topological groups and which have certain algebraic and topological properties.

Introduction. Let X be a metric space and X^* be the space of all Lebesgue measurable functions from $[0, 1]$ into X with the topology of convergence in measure. The space X^* is a contractible, locally contractible metric space in general. If X has a complete metric, then X^* also has a complete metric. Bessaga and Pełczyński [3] have shown that if X is $\{0, 1\}$, $[0, 1]$, or R, then X^* is homeomorphic to l_2, separable infinite-dimensional Hilbert space. More recently they have shown [4] that X^* is homeomorphic to l_2 whenever X is a complete separable metric space with more than one point. This fact is used in this paper to show the existence of topological groups which are connected separable Fréchet manifolds and which have certain algebraic and topological properties. The construction we use is a modification of one used by Hartman and Mycielski [8]. Primarily we study two types of topological groups: periodic topological groups and those topological groups whose underlying group structure is isomorphic to the additive reals. The results are contrasted with the structure theory of locally compact topological groups [10 and 16] and with the theory of infinite-dimensional Lie groups [14].

In the last section of the paper it is shown that if π is any abelian group, then π is the fundamental group of a completely metrizable topological group G_π which is a $K(\pi, 1)$. If π is countable, then G_π is a separable Fréchet manifold. If π is any group, then we show the existence of a completely metrizable homogeneous space which is a $K(\pi, 1)$. If π is countable, then the space will be