

RATIONAL WHITEHEAD PRODUCTS AND A SPECTRAL SEQUENCE OF QUILLEN

CHRISTOPHER ALLDAY

A third order rational Whitehead product is defined in terms of the appropriate differential graded Lie algebra. The product is used to calculate the second differential in Quillen's rational reverse Adams spectral sequence. Some facts about a fourth order product are stated, and conjectures are made concerning higher order products. The products of this paper are compared to those defined by Zeeman, Hardie, and Porter.

In the paper "Rational homotopy theory" [7], Quillen introduces a very interesting rational unstable reverse Adams spectral sequence. He also introduces a functor, which assigns to each one-connected topological space a differential graded Lie algebra, whose homology is the rational homotopy Lie algebra of the space. In this paper we show, first, how higher order rational Whitehead products may be defined using the DG Lie algebra, secondly, how these Whitehead products may be used to calculate the differentials in Quillen's spectral sequence, and, thirdly, how these Whitehead products compare to those defined by Zeeman, Hardie, Porter, et al.

In the first section we review some of the notation and material of [7]. In the second section we define the third order rational Whitehead product, and show how it and the ordinary (second order) rational Whitehead product fit into Quillen's spectral sequence. Further, we indicate how the higher order products may be defined, but an explicit definition is included only for the fourth order product. In the final section a theorem is established, which provides a mechanism for comparing our rational products with those defined classically, and this comparison is carried out for the third order product.

1. Review of material and notation from [7]. We are concerned with the following three categories: $\mathcal{T}_2$, the category of 1-connected pointed topological spaces, and basepoint preserving continuous maps, whose weak equivalences are those maps which induce isomorphisms on rational homotopy.

$(DGL)_1$, the category of reduced differential graded Lie algebras over $\mathbb{Q}$ and DG Lie algebra morphisms, whose weak equivalences are those morphisms which induce isomorphisms on homology.

$(DGC)_2$, the category of 2-reduced differential graded coalgebras over