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# ONE SIDED PRIME IDEALS

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**Consider a right ideal $L$ in a ring (with $1 \in R$ or $1 \notin R$), its idealizer $N = \{n \in R \mid nL \subseteq L\}$, the bound $P = \{r \in L \mid Rr \subseteq L\} \lhd R$ of $L$, and the ideal $H = \{n \in N \mid nL \subseteq P\} \lhd N$. II. Some of the ideal structure of the ring $N/P$ is determined for a class of one sided prime ideals $L$ more general than the almost maximal ones and without any chain conditions on $R$ (Theorem II). III. When $L \neq P$ the following conditions are necessary and sufficient for $N/P$ to have precisely two unequal, nonzero minimal prime ideals $L/P$ and $H/P$:**

**(i) $H \neq P$;**

**(ii) $L/P < R/P$ is not essential;**

**(iii) $L/P$ is a maximal annihilator in $R/P$;**

**(iv) the left annihilator of $L/P$ is not zero;**

**(v) $L = \{r \in R \mid ur \in P\}$ for some $u \in N \backslash L$ (Theorem III).**

Much of the theory of primitive rings arising from a regular maximal right ideal has been generalized to an almost maximal right ideal $L$ in a ring $\boldsymbol{R}$ ([6], [7], and [8]). Thus if $P \subsetneqq L$ is the biggest ideal of $\boldsymbol{R}$ inside $L$, then $\boldsymbol{R}/P$ is called almost primitive. Recently, the Krull-dimension of modules has received some attention ([9], [11], and [12]), and has also been considered for ordinals rather than integers ([5]). In order to show how the results of [4] are a special case of the present development, the above two apparently distinct concepts-almost maximal one sided ideals and the Krull-dimension-are related in Theorem I. Some of the ideal structure of the rings $\boldsymbol{R}/\boldsymbol{P}$ and $N/P$ for $L$ almost maximal can be obtained as a special case of a more general result (Theorem II). The latter shows that either (0) is the unique minimal prime ideal of $N/P$, or it has two distinct minimal primes $0 \neq H/P$ and $0 \neq L/P$. If $L \neq P$ and $\boldsymbol{L} < \boldsymbol{R}$ is not essential, then the last alternative holds. However, necessary and sufficient conditions for the latter to hold have to be phrased in terms of the quotient ring $\boldsymbol{R}/P$ (Theorem III). Finally, to see whether some of the results are best possible, some examples and counterexamples are constructed. However, they fail to show that Theorem II is best possible, and this still remains an open question.

## 1. Preliminaries.

Various types of modules and one sided prime ideals are defined.

NOTATION 1.1. For any ring $\boldsymbol{R}$, define $\boldsymbol{R}^1$ as $\boldsymbol{R}^1 = \boldsymbol{R}$ in case $R$ has an identity; otherwise, $\boldsymbol{R}^1 = \boldsymbol{Z} \times \boldsymbol{R}$ is the ring with an identity