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ON A "SQUARE" FUNCTIONAL EQUATION

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In a recent paper Stanton and Cowan have generalized the Pascal's triangle to a tableau. They have developed several expressions for these numbers, using combinatorial techniques. In the present paper we derive some of their results very simply, by using the calculus of finite differences. We further obtain the relations of these numbers to hypergeometric function and derive many relations among these numbers which are useful in constructing the tableau.

1. Introduction. The triangular array of binomial coefficients, well-known as Pascal's triangle, has been much studied. Basically, it depends on the recursion relation

$$f(n+1, r) = f(n, r) + f(n, r-1) . \tag{1}$$

In a recent paper Stanton and Cowan [5], have considered a generalization of this situation by defining a tableau by the recurrence relation

$$g(n+1, r+1) = g(n, r+1) + g(n+1, r) + g(n, r) . \tag{2}$$

This formula, together with the boundary conditions, $g(n, 0) = g(0, r) = 1$, uniquely determines $g(n, r)$. The lower half of the first portion of this tableau is presented in Table 1, the upper half can be obtained by symmetry in n and r (see § 2).

TABLE 1

$g(n, r)$							
1							
1	3						
1	5	13					
1	7	25	63.				
1	9	41	129	321			
1	11	61	231	681	1683		
1	13	85	377	1289	3653	8989	
1	15	113	575	2241	7183	19825	48639

Stanton and Cowan [5], have developed several expressions for these numbers $g(n, r)$, and indicated that they have a further com-