PACIFIC JOURNAL OF MATHEMATICS
Vol. 51, No. 2, 1974

ON A SPLITTING FIELD OF REPRESENTATIONS OF A FINITE GROUP

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The theorem of P. Fong about a splitting field of representations of a finite group G will be improved to the effect that the order of G mentioned in it will be replaced by the exponent of G. The proof depends on the Brauer-Witt theorem and properties of cyclotomic algebras.

Let Q denote the rational field. For a positive integer n, ζ_n is a primitive nth root of unity. Let χ be an irreducible character of a finite group G (an irreducible character means an absolutely irreducible one). Let K be a field of characteristic 0. Then $m_K(\chi)$ denotes the Schur index of χ over K. The simple component of the group algebra $K[G]$ corresponding to χ is denoted by $A(\chi, K)$. Its index is exactly $m_K(\chi)$. If L/K is normal, $\mathscr{G}(L/K)$ is the Galois group of L over K.

In this paper we will prove the following:

THEOREM. *Let G be a finite group of exponent $s = l^a n$, where l is a rational prime and $(l, n) = 1$. Let $k = Q(\zeta_n)$ if l is odd, let $k = Q(\zeta_n, \zeta_4)$ if $l = 2$. Then, $m_k(\chi) = 1$ for every irreducible character χ of G.*

REMARK. In Fong [2, Theorem 1], the above s denoted the order of G (instead of the exponent of G).

First we review

BRAUER-WITT THEOREM. *Let χ be an irreducible character of a finite group G of exponent s. Let q be a prime number. Let K be a field of characteristic 0 with $K(\chi) = K$. Let L be the subfield of $K(\zeta_s)$ over K such that $[K(\zeta_s): L]$ is a power of q and $[L: K] \not\equiv 0 \pmod q$. Then there is a subgroup F of G and an irreducible character ξ of F with the following properties:* (1) *there is a normal subgroup N of F and a linear character ψ of N such that $\xi = \psi^F$ and $L(\xi) = L$,* (2) *$F/N \cong \mathscr{G}(L(\psi)/L)$,* (3) *$m_L(\xi)$ is equal to the q-part of $m_K(\chi)$,* (4) *for every $f \in F$ there is a $\tau(f) \in \mathscr{G}(L(\psi)/L)$ such that $\psi(fnf^{-1}) = \tau(f)(\psi(n))$ for all $n \in N$, and* (5) *$A(\xi, L)$ is isomorphic to the crossed product $(\beta, L(\psi)/L)$ where, if S is a complete set of coset representatives of N in F $(1 \in S)$ with $ff' = n(f, f')f''$ for $f, f', f'' \in S$, $n(f, f') \in N$, then $\beta(\tau(f), \tau(f')) = \psi(n(f, f'))$.*