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ON MATRIX MAPS OF ENTIRE SEQUENCES

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In this note the linear space E of entire sequences and various subspaces are considered. The fact that E represents the space of entire functions is emphasized by determining subspaces in terms of order and type. Matrix maps between the subspaces are characterized, and a related result and an example are also given.

1. Subspaces of E determined by order. First we recall that if $M(r) = \max |f(z)|$ on the circle $|z| = r$, then the order of the entire function f is $\rho = \limsup [(\log\log M(r))/\log r]$, and its type is $\tau = \limsup [(\log M(r))/r^{\rho}]$, assuming $\rho < \infty$. If $f(z) = \sum_0^\infty x_n z^n$ is an entire function, then it has finite order ρ if and only if $\mu = \limsup_n [n \log n/\log(1/|x_n|)]$ is finite, and then $\rho = \mu$([1], p. 9).

DEFINITION 1.1. We say the complex sequence $x = \{x_n\}_0^\infty$ is analytic if the corresponding power series $\sum x_n z^n$ has radius of convergence $r(x) > 0$. x is an entire sequence if $r(x) = \infty$, and its order and type are those of the power series.

DEFINITION 1.2. For each $\rho \in [0, \infty)$, let $O(\rho)$ be the set of entire sequences of order not exceeding ρ. For each $\rho \in (0, \infty]$, let $O'(\rho)$ be the set of entire sequences of order less than ρ.

DEFINITION 1.3. If $0 \leqq \rho < \infty$, let ρ^+ be the class of real sequences $\{\rho_n\}_1^\infty$ such that $\rho_n \searrow \rho$ and $\rho_n > \rho$. If $0 < \rho \leqq \infty$, let ρ^- be similarly defined, but with $\rho_n \nearrow \rho$ and $0 < \rho_n < \rho$.

DEFINITION 1.4. Let $\alpha = \{\alpha_n\}_0^\infty$ be a complex sequence with no zero-terms, and let $s(\alpha) = \{\text{complex } x \mid \alpha_n x_n \to 0\}$.

If we define $\|x\|_\alpha = \sup |\alpha_n x_n|$, then $(s(\alpha), \|\cdot\|_\alpha)$ is a BK space ([8], Satz 5.4).

We will now characterize those matrices $A = (a_{nk})$ which map $s(\alpha) \to s(\beta)$. If f is a continuous linear functional on $s(\alpha)$, then f can be represented in the form $f(x) = \sum c_n \alpha_n x_n$, where $\sum |c_n| < \infty$ ([8], Satz 5.4). It is easily shown that the coefficients c_n in this representation are unique, and that $\|f\| = \sum |c_n|$. Suppose A maps $s(\alpha) \to s(\beta)$. Define $f_n(x) = \beta_n \sum_k a_{nk} x_k$. It is known ([7], Corollary 5, p. 204, or [8], Satz 4.4) that a matrix map between FK spaces is continuous, and $f_n = \beta_n P_n \circ A$ (where P_n is the nth projection map), so f_n is a continuous linear functional on $s(\alpha)$ with norm $\|f_n\| = \sum_k |\beta_n a_{nk}/\alpha_k|$.