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ORTHOGONAL GROUPS OF DYADIC UNIMODULAR QUADRATIC FORMS II

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Let $O(M)$ be the orthogonal group of a unimodular quadratic form over the integers in a dyadic local field. The subgroups of $O(M)$ normalized by the commutator subgroup are classified when the rank $r(M) \geqq 9$, or when $r(M) \geqq 7$ and the residue class field has at least 8 elements.

Classifications of the subgroups of an orthogonal group normalized by the commutator subgroup have been given by many authors. For isotropic nonsingular quadratic forms over fields there is the fundamental result of Dickson [3] and Dieudonné [4]: The projective commutator subgroup is simple when the form has dimension at least 5. Other proofs of this, which allow the field to have characteristic two, have been given by Eichler [5] and Tamagawa [17]. In [12], Klingenberg generalized this result to nondegenerate quadratic forms over local rings, provided the residue class field is not of characteristic two, and classified the subgroups normalized by the commutator subgroup by using congruence subgroups and mixed commutator subgroups. Klingenberg's work has been further extended in [1, 2, 7-10, 13, 16, 18, 19] by relaxing the restrictions either on the form or on the ring. In particular, I studied this problem for unimodular quadratic forms over the ring of integers in a dyadic local field with 2 an unramified prime and the residue class field having at least 8 elements [9, 10]. These last two restrictions will now be removed, that is, 2 may ramify and there is no restriction on the residue class field (except only that it is perfect).

An outline of the paper follows. Denote by $\mathfrak{o}$ the ring of integers in a dyadic local field F and by M a free $\mathfrak{o}$-module of finite rank $r(M) \geqq 3$ endowed with an isotropic symmetric bilinear form $B: M \times M \rightarrow \mathfrak{o}$ with determinant a unit in $\mathfrak{o}$. After introducing some basic isometries, the commutator subgroup $\Omega(M)$ of the orthogonal group $O(M)$ is determined. Apart for a few exceptional modules M with small rank, $\Omega(M)$ is equal to the spinorial kernel of $O(M)$ and is generated by the Siegel transformations. Next, the "primitive" submodules M_ξ, $\xi \in \Xi$(Ξ a suitable indexing set), invariant under the action of the commutator subgroup are determined. For each ideal $\mathfrak{a}$ in $\mathfrak{o}$, the submodules $\mathfrak{a}M_\xi$ are still invariant and are used to define the subgroups $\mathscr{G}(\mathfrak{a}M_\xi)$ and $\mathscr{F}(\mathfrak{a}M_\xi)$. The main result is:

If $r(M) \geqq 9$, a subgroup $\mathscr{N}$ of the orthogonal group $O(M)$ is