PACIFIC JOURNAL OF MATHEMATICS
Vol. 52, No. 2, 1974

PRECOMPACT AND COLLECTIVELY SEMI-PRECOMPACT SETS OF SEMI-PRECOMPACT CONTINUOUS LINEAR OPERATORS

ANDREW S. GEUE

A mapping f from a set B into a uniform space $(Y, \mathscr{V})$ is said to be precompact if and only if its range $f(B) = \{f(b): b \in B\}$ is a precompact subset of Y. The precompact subsets of $\mathscr{K}(B, Y)$, the set of all precompact mappings from B into Y with its natural topology of uniform convergence, are characterized by an Ascoli-Arzelà theorem using the notion of equal variation.

A linear operator $T: X \rightarrow Y$, where X and Y are topological vector spaces, is said to be semi-precompact if $T(B)$ is precompact for every bounded subset B of X. Let $\mathscr{L}_{\mathfrak{b}}[X, Y]$ denote the set of all continuous linear operators from X into Y with the topology of uniform convergence on bounded subsets of X. Let $\mathscr{K}_{\mathfrak{b}}[X, Y]$ denote the subspace of $\mathscr{L}_{\mathfrak{b}}[X, Y]$ consisting of the semi-precompact continuous linear operators with the induced topology. The precompact subsets of $\mathscr{K}_{\mathfrak{b}}[X, Y]$ are characterized. A generalized Schauder's theorem for locally convex Hausdorff spaces is obtained. A subset $\mathscr{H}$ of $\mathscr{L}[X, Y]$ is said to be collectively semi-precompact if $\mathscr{H}(B) = \{H(b): H \in \mathscr{H}, b \in B\}$ is precompact for every bounded subset B of X. Let X and Y be locally convex Hausdorff spaces with Y infrabarrelled. In § 5 the precompact sets of semi-precompact linear operators in $\mathscr{L}_{\mathfrak{b}}[X, Y]$ are characterized in terms of the concept of collective semi-precompactness of the sets and certain properties of the set of adjoint operators.

1. Introduction. Let X and Y be topological vector spaces over the field of complex numbers C and $\mathscr{L}[X, Y]$ the set of continuous linear operators from X into Y. For a subset $\mathscr{H} \subset \mathscr{L}[X, Y]$ and a subset B of X, let $\mathscr{H}(B) = \{H(b): H \in \mathscr{H}, b \in B\}$.

DEFINITION 1.1. A linear operator $T: X \rightarrow Y$ is said to be *precompact* (*compact*) if there exists a neighborhood V of zero in X such that $T(V)$ is precompact (relatively compact). A linear operator $T: X \rightarrow Y$ is said to be *semi-precompact* (*semi-compact*) if $T(B)$ is precompact (relatively compact) for every bounded subset B of X.

The latter terminology is that of Deshpande and Joshi [14] and coincides with the term "boundedly precompact" used by Ringrose [27]. Clearly, precompactness of an operator is a much stronger