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RECONSTRUCTING INFINITE GRAPHS

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It is a well-known conjecture of S. M. Ulam that any finite graph of order at least three can be reconstructed from its maximal vertex-deleted subgraphs. Formally (writing G_v for $G - v$) *Ulam's Conjecture* states: if G and H are finite graphs of order at least three such that there is a bijection $\sigma: V(G) \rightarrow V(H)$ with the property

$$(1) \qquad G_v \cong H_{\sigma(v)} \text{ for all } v \in V(G) ,$$

then $G \cong H$. This conjecture has not been proved in general, although it was shown by P. J. Kelly to be true for disconnected graphs and trees and has also been verified for several other classes of graphs. The purpose of this paper is to examine Ulam's Conjecture for infinite graphs. (It is trivial to determine, from any G_v, whether or not a graph G is infinite.) Results are obtained which can loosely be viewed as extensions of Kelly's work on disconnected graphs and trees.

In §2 it is shown that infinite graphs G and H satisfying (1) must have the same finite components, occurring with the same multiplicity. Corollaries of this are that if G either has only finite components, or has some finitely occurring finite component, then $G \cong H$. In §3 the conjecture is proved for m-coherent locally finite trees, where m is finite and greater than one. This furnishes a partial solution to the reconstruction problem for infinite trees, raised by C. St. J. A. Nash-Williams.

We have used the language of reconstruction in our proofs. However, it should be noted that the results are existential in nature and not algorithmic. Throughout the paper G and H will denote infinite graphs satisfying condition (1) of Ulam's Conjecture. Any notation and terminology not defined can be found in Harary [3].

2. Disconnected graphs.

We denote by $c(G)$ the number of components of G, and by $c(G; K)$ the number of components of G that are isomorphic to K. A finite connected graph J is called a *K-producer* if $c(J_v; K) > 0$ for all $v \in V(J)$. (Since J has a non-cut-vertex, J must be regular and of order one more than the order of K; hence K determines J up to isomorphism.) An *endvertex of* G is a vertex of degree one.

LEMMA 2.1. *If L is infinite and connected and K is finite, then there is an infinite set $S \subseteq V(L)$ such that $c(L_v; K) = 0$ for all $v \in S$.*