PACIFIC JOURNAL OF MATHEMATICS
Vol. 53, No. 1. 1974

STRONGLY BOUNDED OPERATORS

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Recently Dobrakov, Batt and Berg, and Brooks and Lewis have studied the class of strongly bounded operators on continuous function spaces in detail. In many cases, these operators coincide with the weakly compact operators and therefore form a norm-closed two-sided ideal; and it is known in general that the strongly bounded operators form a norm-closed left ideal. In this note an example is presented which shows that the strongly bounded operators do not form a right ideal.

Let each of E and F denote a B-space, let H denote a locally compact Hausdorff space, and let $\boldsymbol{B} \equiv C_0(H, E)$ be the B-space (under sup norm) of all continuous E-valued functions defined on H which vanish at infinity. It is known that if an operator (=continuous linear transformation) L maps $\boldsymbol{B}$ into F and $\Sigma = \Sigma(H)$ is the Borel σ-algebra of subsets of H, then there is a unique weakly regular finitely additive representing measure $m: \Sigma \to B(E, F^{**})$ so that $L(f) = \int f dm$, $f \in \boldsymbol{B}$, e.g., see Batt and Berg [1] and Brooks and Lewis [3] for further details.

The representing measure $m: \Sigma \to B(E, F^{**})$ is said to be *strongly bounded* (s-bounded) provided that if (A_i) is a disjoint sequence, then $\tilde{m}(A_i) \to 0$, where $\tilde{m}$ denotes the semivariation of m; an operator will be called s-bounded if its representing measure is s-bounded. Equivalent formulations are given in the following lemma, which we state for reference purposes. For the details of the proof, one may consult Brooks and Lewis [3].

LEMMA 1. *Suppose that m is a representing measure, $m \leftrightarrow L$. Then the following are equivalent*:

(a) *m is s-bounded;*

(b) *$\tilde{m}(A_i) \to 0$ whenever $A_i \searrow \varnothing$;*

(c) *$\{|m_z|: z \in F_1^*\}$ is conditionally weakly compact in $ca(\Sigma, E^*)$;*

(d) *$\Sigma m(A_i)x_i$ converges (in F) for each disjoint sequence (A_i) and $(x_i) \subset E_1$ (=closed unit ball of E);*

(e) *If $(A_i) \subset \Sigma$ and $A_i \searrow \varnothing$, then there is a nested sequence U_n of open sets so that $A_n \subset U_n$ and $L(f_n) \to 0$ uniformly for each sequence (f_n) so that support $(f_n) \subset U_n$, $||f_n|| \leqq 1$.*

The class of s-bounded operators which we denote by S has been studied in [1], [2], [3], and [5], with the sharpest results being