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BRANCHED IMMERSIONS ONTO COMPACT ORIENTABLE SURFACES

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In this paper smooth maps $f: M^n \to N^n$ with a zero-dimensional critical set are considered. The singularities of these maps in the case $n = 2$ are known to be points where f is locally topologically equivalent to $z \to z^d$ ($d = 2, 3, \cdots$). Originally these singularities were studied in connection with the regularity of Douglas' solution to Plateau's problem.

In § 1, an Euler characteristic formula is developed which generalizes both the Riemann-Hurwitz equation from complex analysis and the usual Euler characteristic formula for covering maps. Section 2 is devoted to several technical lemmas while § 3 applies these lemmas to the case where M is the disc (with holes) and N is a compact orientable 2-manifold. It is shown that for the existence of such a map there is a lower limit depending upon the genus of N and on the number of holes of M.

The singularities of these maps have been characterized by Church and Timourian [2]. In the case $n = 2$ these maps are locally topologically equivalent to $z \to z^d$ ($d = 2, 3, \cdots$) and for $n > 2$, these maps are covering projections. For $n = 2$, the singularities and maps are special cases of branch points and branched immersions. In this case, the Euler characteristic formula represents a generalization of the classical Riemann-Hurwitz equation for light interior transformations on 2-manifolds. When $f^{-1}(\partial N) = \partial M$, the formula produced here reduces to the Riemann-Hurwitz equation. For $n > 2$, the maps are covering projections and the Euler characteristic formula reduces to the usual equation.

The mappings considered in this paper are not, in general, interior transformations on the boundary of M. These considered here, however, appear to have more applications with regard to questions which have arisen from Plateau's problem. Heinz [6] and Gulliver, Osserman, and Royden [5] have shown that these maps are of much more interest than solely in the context of minimal surfaces.

A very readable account of the Riemann-Hurwitz formula may be found in Whyburn [10]. The authors wish to express their gratitude to the referee for his many helpful suggestions.

1. Let $f: M^n \to N^n$ be a continuous map between orientable n-manifolds with or without boundary having finite fibres. If $\partial M \neq$