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CONTINUOUS SPECTRA OF A SINGULAR SYMMETRIC DIFFERENTIAL OPERATOR ON A HILBERT SPACE OF VECTOR-VALUED FUNCTIONS

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Let H be the Hilbert space of complex vector-valued functions $f: [a, \infty) \to C^2$ such that f is Lebesgue measurable on $[a, \infty)$ and $\int_a^\infty f^*(s)f(s)ds < \infty$. Consider the formally self adjoint expression $\iota(y) = -y'' + Py$ on $[a, \infty)$, where y is a 2-vector and P is a 2×2 symmetric matrix of continuous real valued functions on $[a, \infty)$. Let D be the linear manifold in H defined by

$$D = \{f \varepsilon H: f, f' \text{ are absolutely continuous on compact subintervals of } [a, \infty),\ f \text{ has compact support interior to } [a, \infty) \text{ and } \iota(f) \varepsilon H\}\,.$$

Then the operator L defined by $L(y) = \iota(y)$, $y \varepsilon D$, is a real symmetric operator on D. Let L_0 be the minimal closed extension of L. For this class of minimal closed symmetric operators this paper determines sufficient conditions for the continuous spectrum of self adjoint extensions to be the entire real axis. Since the domain, D_0, of L_0 is dense in H, self adjoint extensions of L_0 do exist.

A general background for the theory of the operators discussed here is found in [1], [3], and [5]. The theorems in this paper are motivated by the theorems of Hinton [4] and Eastham and El-Deberky [2]. In [4], Hinton gives conditions on the coefficients in the scalar case to guarantee that the continuous spectrum of self adjoint extensions covers the entire real axis. Eastham and El-Deberky [2] study the general even order scalar operator.

DEFINITION 1. Let $\tilde{L}$ denote a self adjoint extension of L_0. Then we define the *continuous spectrum*, $C(\tilde{L})$, of $\tilde{L}$ to be the set of all λ for which there exists a sequence $\langle f_n \rangle$ in $D_{\tilde{L}}$, the domain of $\tilde{L}$, with the properties:

(i) $\|f_n\| = 1$ for all n,

(ii) $\langle f_n \rangle$ contains no convergent subsequence (i.e., is not compact), and

(iii) $\|(\tilde{L} - \lambda)f_n\| \to 0$ as $n \to \infty$.

For the self adjoint operator $\tilde{L}$ we have the following well-known lemma.

LEMMA 1. *The continuous spectrum of $\tilde{L}$ is a subset of the real numbers.*