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INVARIANT SUBSPACES OF COMPACT OPERATORS ON TOPOLOGICAL VECTOR SPACES

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Let (H, τ) be a topological vector space and let T be a compact linear operator mapping H into H (i.e., $T[V]$ is contained in a τ-compact set for some τ-neighborhood V of the zero vector in H). Sufficient conditions are given for (H, τ) so that T has a non-trivial, closed invariant linear subspace. In particular, it is shown that any complete, metrizable topological vector space with a Schauder basis satisfies the conditions stated in this paper. The proofs and conditions are stated within the framework of nonstandard analysis.

Introduction. This paper considers the following problem: given a compact operator T (Definition 2.11) on a topological vector space (H, τ), does there exist a closed nontrivial linear subspace F of H such that $T[F] \subset F$? Aronszajn and Smith gave an affirmative answer to the above question when H is a Banach space (see **[1]**). Also it is easily shown that the Aronszajn and Smith result can be extended to locally convex spaces. However, it appears that other methods must be used for nonlocally convex spaces.

Sufficient conditions are given for a topological vector space so that a compact linear operator defined on the space has at least one nontrivial closed invariant linear subspace (Definitions 2.1 and 4.1, Theorems 3.2, 4.2 and 4.7). In particular, it is shown that a Fréchet space with a Schauder basis satisfies the conditions given in this paper (Theorem 5.6). The basic techniques are an outgrowth of Bernstein's and Robinson's methods in **[2]**, **[3]** and **[4]**; consequently, the proofs and arguments of the main results are stated within the framework of nonstandard analysis. It will be assumed that the reader is familiar with Abraham Robinson's book on Nonstandard Analysis **[11]** or W. A. J. Luxemburg's paper on Monad Theory **[10]**.

1. Nonstandard topological vector spaces. Let us briefly examine the basic concepts of nonstandard topological vector space theory that are used in this paper. A more detailed discussion of nonstandard topological vector spaces can be found in **[6]** and **[7]**. Let $\mathbf{C}$ be the complex numbers, let H be a vector space over $\mathbf{C}$ and let B_Γ be the full set theoretical structure of $H \cup \mathbf{C}$. Throughout this paper (H, τ) will denote a *complex* topological vector space H with topology