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DISTRIBUTIONAL BOUNDARY VALUES IN THE DUAL SPACES OF SPACES OF TYPE $\mathcal{S}$

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In this paper it is shown that elements of a space of analytic functions defined in the tube domain $T^C = \mathbf{R}^n + iC$, where C is an open convex cone of a certain type, obtain distributional boundary values in the weak topology of the distribution spaces $(\mathcal{S}^\alpha)'$, $\alpha = (\alpha_1, \cdots, \alpha_n)$, $\alpha_j \geqq 1$, $j = 1, \cdots, n$; and representation results of the analytic functions in terms of the boundary values are given. Converse results are obtained in which an analytic function in the defined space is constructed from a given distribution in $(\mathcal{S}^\alpha)'$, and some applications of the distributional boundary value theorems are obtained. The main results are proved with the aid of several new lemmas concerning the C^∞ function spaces of type $\mathcal{S}$ and their dual spaces. The results obtained here are motivated by known results used in the construction of local fields in quantum field theory.

1. Introduction. Gel'fand and Shilov [9] have introduced the spaces of type $\mathcal{S}$, which have been shown to be of importance in at least two areas of applications. Gel'fand and Shilov [10] have used these spaces for a study of the Cauchy problem; while Constantinescu [4] and Rieckers [11] have used them in their studies of quantum field theory.

Constantinescu [4] constructs local fields, which are a category of fields larger than the strictly localizable ones, and proves that the vacuum expectation values in a local field theory are boundary values of functions analytic in a tube domain corresponding to the forward light cone. These vacuum expectation values are in fact distributional boundary values of the analytic functions in the weak topology of the dual spaces of the spaces of type $\mathcal{S}$ and are distributions in these dual spaces.

The present paper is motivated by the distributional boundary value results of Constantinescu [4, Theorems 1 and 2]. We shall generalize these results to functions analytic in tube domains corresponding to open convex cones. The quantum mechanical setting of the tube domain corresponding to the light cone is not the only special case of our generalizations; we shall see that the results corresponding to [4, Theorems 1 and 2] for the important mathematical settings of the half plane in $\mathbf{C}^1$ and octant in $\mathbf{C}^n$ are also special cases of the theorems we obtain here. We not only obtain generalizations of the known