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CLOSE-TO-STARLIKE HOLOMORPHIC FUNCTIONS OF SEVERAL VARIABLES

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Let X be a finite dimensional complex normed linear space with unit ball $B = \{x \in X: \|x\| < 1\}$. In this paper the notion of a close-to-starlike holomorphic mapping from B into X is defined. The definition is a direct generalization of W. Kaplan's notion of one dimensional close-to-convex functions. It is shown that close-to-starlike mappings of B into X are univalent and these mappings are given an alternate characterization in terms of subordination chains.

1. **Introduction.** In 1952 [2] W. Kaplan defined the class of close-to-convex functions: $f(z) = z + \cdots$ analytic and

$$Re\,\{f'(z)/\phi'(z)\} > 0 \tag{1.1}$$

in $|z| < 1$, for some univalent convex function $\phi(z) = az + \cdots$ ($|z| < 1$). Subsequent interest in this class stems from Kaplan's observation that (1.1) implies $f(z)$ is univalent in $|z| < 1$. In this paper we present the natural generalization of close-to-convex vector valued functions in finite dimensional complex spaces. This is a continuation of recent work on vector valued holomorphic starlike and convex mappings [7], [8]. We use the notions of subordination chains of holomorphic maps in C^n and the generalized Loewner differential equation [5] to elucidate the geometry of the mappings.

2. **Statement of main results.** Let X be a finite dimensional complex normed linear space with dual X^* and $\mathscr{L}(X)$ the set of continuous linear operators from X into X. We let $\mathscr{H}(B)$ denote the set of functions $f(x)$ that are holomorphic in the unit ball $B = \{x \in X: ||x|| < 1\}$ with values in X. The notation $f(x) = ax + \cdots, a \in C$, for $f \in \mathscr{H}(B)$ indicates that $Df(0) = aI$ where I is the identity in $\mathscr{L}(X)$.

For $0 \neq x \in X$ we define

$$T(x) = \{x^* \in X^*: x^*(x) = ||x|| \quad \text{and} \quad ||x^*|| = 1\}\,,$$

and note that $T(x)$ is nonempty by the Hahn-Banach theorem. We let $\mathscr{M}$ denote the class of functions $h(x) = x + \cdots \in \mathscr{H}(B)$ such that $Re\, x^*(h(x)) > 0$ for each $x \in B - \{0\}$ and $x^* \in T(x)$. A mapping $g(x) = x + \cdots \in \mathscr{H}(B)$ is called starlike if g is univalent in B and $tg(B) \subset g(B)$ for all $0 \leqq t \leqq 1$.