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COUNTABLE PRODUCTS OF GENERALIZED COUNTABLY COMPACT SPACES[1]

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In this paper two ways of generalizing compactness are studied. We may consider various types of refinements of open covers, such as countable open refinements, locally finite open refinements, etc. In another direction, countably compact spaces may be characterized as having the property that any sequence has a cluster point. Spaces which require that certain sequences have cluster points, such as Σ-spaces, $w\Delta$-spaces, and q-spaces, will be referred to as generalized countably compact spaces.

These more general properties do not behave as well as compactness with respect to products. For example, the product of two Lindelöf spaces need not even be meta-Lindelöf, and the product of two countably compact spaces need not be a q-space. The question naturally arises as to what conditions must be placed on such spaces to insure that they are better behaved with respect to products.

Let Q be a class of generalized countably compact spaces, let $X_1, X_2, \cdots$ be a sequence of spaces each of which belongs to Q. Consider the following two questions.

1. When does $\prod_{n=1}^{\infty} X_n$ belong to Q?

2. Suppose that each X_n has a covering property P which generalizes compactness. When does $\prod_{n=1}^{\infty} X_n$ have P?

In § 3 we answer the first question where Q is any of the following classes: countably compact spaces, Σ-spaces, $w\Delta$-spaces, q-spaces, β-spaces, and wN-spaces. In § 4 the second question is answered for the case where Q is the class of $w\Delta$-spaces and P is one of the following: paracompact, metacompact, subparacompact, and meta-Lindelöf.

2. **Preliminaries.** Unless otherwise stated, no separation axioms will be assumed. Undefined terms are used as defined in [16], except that paracompact spaces are always Hausdorff. The set of natural numbers will be denoted by N, and i, j, k, and n will denote elements of N. If $\mathscr{A}_1, \cdots, \mathscr{A}_n$ are collections of subsets of a set X, we let $\bigwedge_{i=1}^{n} \mathscr{A}_i$ denote the collection $\{\bigcap_{i=1}^{n} A_i \mid A_i \in \mathscr{A}_i, i = 1, \cdots, n\}$. A sequence $\mathscr{A}_1, \mathscr{A}_2, \cdots$ of collections is said to be *decreasing* if $\mathscr{A}_{n+1}$ refines $\mathscr{A}_n$ (written $\mathscr{A}_{n+1} < \mathscr{A}_n$), for $n = 1, 2, \cdots$. Also, if $\mathscr{A}$ is a

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