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MULTIPLICATIVE OPERATIONS IN BP^*BP

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One of the present computational difficulties in complex cobordism theory is the lack of a known algebra splitting of BP^*BP, the algebra of stable cohomology operations for the Brown-Peterson cohomology theory, analogous to the splitting isomorphism

$$MU^*MU \approx MU^*(pt) \otimes S$$

where S is the Landweber-Novikov algebra. S has the added advantage of being a cocommutative Hopf algebra over Z.

This paper does not remove this difficulty, but we will show that the monoid of multiplicative operations in BP^*BP, (i.e. those operations which induce ring endomorphisms on BP^*X for any space X), which we will denote by $\Gamma(BP)$, has a submonoid analogous to the monoid of multiplicative operations in S.

The latter monoid is known (see Morava [3]) to be isomorphic to the group of formal power series $f(x)$ over Z such that $f(0) = 0$ and $f'(0) = 1$ and where the group operation is composition of power series.

For the basic properties of MU^*MU and BP^*BP, see Adams [1], especially §§ 11 and 16.

The main construction of this paper was inspired by the work of Honda ([3]) although none of his results are needed here. I am grateful to Jack Morava for bringing Honda's work to my attention.

Before stating our main result we must review the description of $\Gamma(BP)$ implicit in [1] § 16. An operation $\alpha \in \Gamma(BP)$ is characterized by its action on the canonical generator $z \in BP^*(CP^\infty) \cong \pi_*BP[[z]]$. It is shown that $\alpha(z)$ is a power series $f(z)$ over π_*BP where

$$f^{-1}(z) = z +_\mu t_1 z^p +_\mu t_2 z^{p^2} +_\mu t_3 z^{p^3} +_\mu \cdots \tag{1}$$

where $+_\mu$ denotes the sum in the formal group defined over π_*BP and $t_i \in \pi_*BP$. (This formula appears on page 96 of [1].)

The action of α on π_*BP can be read off from (1). Let $l_n \in \pi_{2p^n-2}BP \otimes Q$ be defined by $\log^{BP} x = \sum_{n=0} l_n x^{p^n}$ (l_n is the m_{p^n-1} of [1]). Then we have

$$\alpha(l_n) = \sum_{0 \leq i \leq n} l_n t_{n-i}^{p^i} \tag{2}$$

and this formula also characterizes α.

In other words $\Gamma(BP)$ is an infinite dimensional affine space over