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GENERALISATION OF A "SQUARE" FUNCTIONAL EQUATION

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Recently the difference equation defining the triangular array of binomial coefficients, known as Pascal's triangle, has been extended to a square functional equation which generates a tableau of numbers. In the present paper these results have been generalised and the generating function for this new set of numbers has been obtained. Several relations among these numbers, which help construct the tableau, are studied. Some further generalisations of these numbers are also given.

1. Introduction. Let I, I^+ and R denote the set of integers, the set of nonnegative integers and the set of real numbers, respectively. The function f defined on the lattice $I^+ \times I^+$ which satisfies the difference equation

$$f(n+1, r) = f(n, r) + f(n, r-1) \tag{1}$$

and is uniquely determined by initial values on $I^+ \times \{0\}$ and $\{0\} \times I^+$ describes the well known triangular array of numbers. This has been generalised by many authors (see Gupta [4], [5], Cadogan [1], Stanton and Cowan [6]). In [5] Gupta has studied the square functional equation

$$g(n+1, r+1) = g(n, r+1) + g(n+1, r) + g(n, r) \tag{2}$$

which together with the boundary conditions $g(n, 0) = g(0, r) = 1, \forall n$, $r \in I^+$ uniquely determines a tableau. However, here we obtain a more general class of functions defined by

$$g\colon I^+ \times I^+ \longrightarrow R$$

satisfying the general square functional equation

$$\begin{aligned} g(n, r) = {} & p_1 g(n-1, r) + p_2 g(n, r-1) \\ & + p_3 g(n-1, r-1) p_i \in R\,, \qquad i = 1, 2, 3\,, \end{aligned} \tag{3}$$

subject to certain initial conditions $g(n, 0) = p_1^n$. It may be noted that $g(n, r)$ is not symmetric in n and r. Also notice that if $p_i = 1$, $i = 1, 2, 3$ this reduces to the case studied earlier (Gupta [5]). In the next section we will give some results for this generalised function $g(n, r)$. However details will be skipped since they are similar to the results in Gupta [5] or Stanton and Cowan [6].