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DOUBLY STOCHASTIC MATRICES WITH MINIMAL PERMANENTS

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A simple elementary proof is given for a result of D. London on permanental minors of doubly stochastic matrices with minimal permanents.

A matrix with nonnegative entries is called *doubly stochastic* if all its row sums and column sums are equal to 1. A well-known conjecture of van der Waerden [3] asserts that the permanent function attains its minimum in Ω_n, the set of $n \times n$ doubly stochastic matrices, uniquely for the matrix all of whose entries are $1/n$. The conjecture is still unresolved.

A matrix A in Ω_n is said to be *minimizing* if

$$\operatorname{per}(A) = \min_{S \in \Omega_n} \operatorname{per}(S).$$

The properties of minimizing matrices have been studied extensively in the hope of finding a lead to a proof of the van der Waerden conjecture.

Let $A(i \mid j)$ denote the submatrix obtained from A by deleting its ith row and its jth colum. Marcus and Newman [3] have obtąined inter alia the following two results.

THEOREM 1. *A minimizing matrix A is fully indecomposable*, i.e.,

$$\operatorname{per}(A(i \mid j)) > 0$$

for all i and j.

In other words, if A is a minimizing $n \times n$ matrix then for any (i,j) there exists a permutation σ such that $j = \sigma(i)$ and $a_{s,\sigma(s)} > 0$ for $s = 1, \cdots, i-1, i+1, \cdots, n$.

THEOREM 2. *If* $A = (a_{ij})$ *is a minimizing matrix then*

$$\operatorname{per}(A(i \mid j)) = \operatorname{per}(A) \tag{1}$$

for any (i,j) *for which* $a_{ij} > 0$.